Laboratory work №1.1

Study of statistical processing of experimental data

Purpose: to get acquainted with the processing of experiments' results and apply it to the calculation of the wire's resistivity.

Theory

Measuring some physical quantity, we get numbers that indicate how many times a unit of measurement is contained in the measured quantity.

Due to the imperfection of measuring instruments, measurement methods and our sense organs, errors inevitably appear during measurements.

The error Δx of measurement is the difference between the value found in the experiment and the true value of the physical quantity:

$$\Delta x = x_f - x_{tr}.$$

The true value of the quantity can not be found, and it is impossible completely avoid the measurement errors. However, using a series of measurements and processing their results, ***we can find the approximate value of the measured quantity and indicate the limit values that it is located between***. This is the meaning of processing the results of the experiment.

Repeating the same measurements repeatedly, you can see that their results are scattered around a certain average. Errors, which change the value and sign from experiment to experiment randomly, without any regularity, are called ***random*** ones. They obey the statistical laws, and therefore their value can be evaluated by the theory of probability.

Suppose that, as a result of n measurements of the physical quantity x, is received $x_1, x_2, x_3, \ldots, x_i, \ldots, x_n$. As the best value for the measured quantity, the ***arithmetical average*** obtained from all the results is taken

$$< x >= \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i.$$

To assess the accuracy of the result of the measured value, the following characteristics are used: the confidence interval and the error limit of the arithmetic average.

We introduce the value $S_{<x>}$, characterizing the possible deviation of the found arithmetic average from the true value. It is called the ***standard (or mean-square) deviation of the mean*** and is equal to

$$S_{<x>} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - < x >)^2}{n(n-1)}}.$$

The square of the standard deviation is called ***variance***:

$$S^2 = D$$

Dispersion is a measure of the deviation of random quantities from the true value of measured quantity. The larger D is, the less accurate is the measurement.

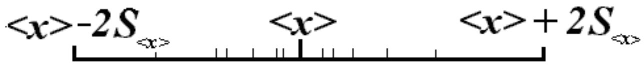
The result can be written in the form

$$x = \langle x \rangle \pm S_{<x>} .$$

Such a form means that the measured quantity x is inside a gap (interval) of width 2 $S_{<x>}$. The interval ($< x > - S_{<x>}, < x > + S_{<x>}$) is shown in the figure.

It is called the ***confidence interval***. This interval contains the true value of the measured quantity with a certain probability. So, the true value of x_{tr} is at the given interval in α = 68% of cases. In this case, α is called the coefficient of confidence or confidence probability. The value of α can be expressed in fractions of unity or %.

If you want to be sure that the x_{tr} is inside the confidence interval, the latter needs to be expanded. If you expand the confidence interval, for example, in 2 times,



the probability that an unknown quantity will be inside this interval increases to α = 95%. Therefore, if the confidence interval increases, the probability that the true value of the quantity is in the considered interval increases, too. However, with the extension of the confidence interval, the absolute and relative measurement error increases.

We considered confidence intervals whose half-widths was $S_{<x>}$ and 2 $S_{<x>}$. Let's consider now a confidence interval whose half-width is equal to $t\, S_{<x>}$.

The standard deviation $S_{<x>}$ is multiplied by a certain number t. This number (called the ***Student's coefficient***) depends on the confidence probability α chosen by the experimenter and the number n of experiments conducted by him.

Student's coefficients $t_{\alpha\, n}$ are calculated in probability theory and are tabulated.

The half-width of the confidence interval is called the ***limiting error*** Δx ***of the arithmetic average***:

$$\Delta x = t_{\alpha,n}\, S_{<x>} .$$

The final result is written in the form

$$x = \langle x \rangle \pm \Delta x = \langle x \rangle \pm t_{\alpha,n}\, S_{<x>} .$$

Table of Student Coefficients $t_{\alpha,n}$.

α \ n	2	3	4	5	6	7	8	9	10
0,8	3,08	1,89	1,64	1,53	1,48	1,44	1,42	1,4	1,38
0,9	6,31	2,92	2,35	2,13	2,02	1,94	1,89	1,86	1,83
0,95	12,7	4,3	3,18	2,78	2,57	2,45	2,36	2,31	2,26
0,98	31,8	6,96	4,54	3,75	3,36	3,14	3	2,9	2,82
0,99	63,7	9,92	5,84	4,6	4,03	3,71	3,5	3,36	3,25

Processing of experimental data

1. **Carry n independent experiments out and determine n values** of the unknown quantity $x_1, x_2, x_3, \dots, x_n$.
2. **Calculate the arithmetic average** of the required value:
$$< x >= \frac{1}{n}\sum_{i=1}^{n} x_i$$
3. **Calculate the deviation of each result from the average:**
$$\Delta x_i = x_i - < x >.$$
4. **Determine the standard deviation of the average**
$$S_{<x>} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - < x >)^2}{n(n-1)}} = \sqrt{\frac{\Delta x_1^2 + \Delta x_2^2 + \dots + \Delta x_n^2}{n(n-1)}}.$$
5. **Set the confidence probability α.** Usually, the confidence probability is equal to 0.90; 0.95; 0.98;. 0.99. From the chosen confidence probability α and completed number of measurements n, determine the Student's coefficient $t_{\alpha,n}$ according to the table.
6. **Calculate the limit error Δx of the arithmetic average** (the half-width of the confidence interval)
$$\Delta x = t_{\alpha,n}\, S_{<x>}\,.$$
7. **Determine the relative error**
$$E = \frac{\Delta x}{< x >} \cdot 100\%.$$
8. **The final result of measurement is written as:**
$$x = < x > \pm \Delta x$$

and indicates the confidence probability α = ...

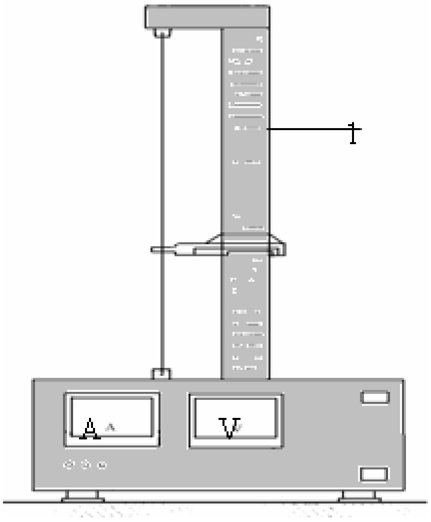
This record means:

as a result of measurements, an average value of $< x >$ with a limit error Δx is found, or otherwise,

with probability α = ... the true value of the measured quantity will lie in the interval from $< x > - \Delta x$ to $< x > + \Delta x$.

We offer a ***measurement of the resistivity of nichrome wire*** as an example of measurements' processing.

The laboratory installation is shown in the figure. The current flows along a vertically stretched wire. Ammeter measures the current and voltmeter does the same for applied voltage.



The resistance R of a constant cross section conductor, made of a homogeneous material, is:

$$R = \rho\frac{l}{S},$$

where ρ - specific resistance; l - length of the conductor; S - cross-sectional area.

The length of the wire l is measured by a measuring scale of the device, the cross-sectional area is calculated determining diameter of the wire d,

$$S = \frac{\pi d^2}{4}.$$

The resistance R can also be determined using the Ohm's law, by measuring the current I and the voltage drop U on the wire by an ammeter and a voltmeter:

$$R = \frac{U}{I}.$$

Therefore, the specific resistance of the wire can be calculated from the formula:

$$\rho = \frac{\pi d^2 U}{4Il}. (1)$$

Performance of work

1. Moving the bracket, set the length l of the wire selected by the instructor.
2. Enable the installation by clicking the "NETWORK" button.
3. Using an ammeter, set the current value I selected by the instructor. Write down the corresponding voltmeter readings.
4. Changing the current, conduct the experiment three to five times.
5. Change the wire length and repeat the same measurements.
6. Write down the measurements in the table.

№	*d*, *m*	*l*, *m*	*I*, *A*	*U*, V	ρ_i, *Ohm*·*m*	<ρ>, *Ohm*·*m*	$\Delta\rho_i$, *Ohm*·*m*	$(\Delta\rho_i)^2$	$S_{<\rho>}$, *Ohm*·*m*	α	$t_{\alpha,n}$	Δρ, *Ohm*·*m*	*E*, %
1.													

2.													
3.													
4.													
5.													
6.													

7. Calculate the specific resistance for each measurement using formula (1) and write down it in the table.
8. Perform mathematical measurement's processing according to the above mentioned technique and write down the final result in the form

$$\rho = (< \rho > \pm \Delta\rho) \text{ Ohm·m with } \alpha = ...$$

Control questions

1. What is the main idea of processing the experimental data? What is called absolute and relative error?
2. What is the meaning of confidence probability and confidence interval?
3. How does the measurement error change with increasing of confidence coefficient?
4. Analyze the Student's coefficients table. How do Student's coefficients change with increasing of experiments number? How does it affect the accuracy of the measurements?
5. What is the sense of record $x = < x > \pm \Delta x$?