3.2.8 Examples of the problems solving

1. An airplane climbs angularly of 30° to the horizontal with an acceleration of $a = 245 m/sec^2$. From the airplane an object dropped out $t_1 = 4 sec$ later after the lifting. Determine: 1) how much longer after dropping τ an object will fall to the ground; 2) the speed v of the object $t_2 = 2 sec$ later after its dropping out of the airplane; 3) its normal a_n and tangential a_τ accelerations when it is falling on the ground.

Solution. 1) We analyze the motion of an object being dropped from an airplane, considering the airplane and the object as material points and we will choose the coordinates origin in the point from which the airplane begins its climb (see figure 3.12). If $t \le t_1$ the object there is at the airplane and moves together with them. Then for this time interval from the definition of the acceleration vector and using the figure 3.12 for object acceleration projections we have

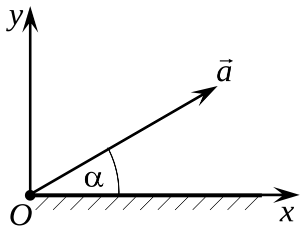

Figure 3.12. To the solution of the problem 3.1

$$\frac{d^2x}{dt^2} = a_x = a \cdot \cos\alpha;$$
$$\frac{d^2y}{dt^2} = a_y = a \cdot \sin\alpha. \qquad (3.49)$$

From the beginning we restrict oneself by the considering of the projection on the axis «*x*». Then after integrating the first of the equations (3.49) we can write

$$v_x(t) = \frac{dx}{dt} = a \cdot \cos\alpha \cdot t + C_1. \qquad (3.50)$$

In given relation *t* is the current variable and C_1 is the integration constant which has the meaning of the initial velocity projection on the axis «*x*». Since $v_x(0) = 0$ from the equality (3.50) we have

$$v_x(0) = C_1 = 0$$

and consequently

$$v_x(t) = a \cdot \cos\alpha \cdot t, \qquad (3.51)$$

if $t \le t_1$. When $t \ge t_1$ for the acceleration projection on the axis «*x*» the next relation takes place

$$\frac{d^2x}{dt^2} = 0, \qquad (3.52)$$

whence $\frac{dv_x}{dt} = 0$ and so

$$v_x(t) = C_2,$$

where C_2 is a new integration constant. We will determinate this constant from the condition $v_x(t_1) = a \cdot \cos\alpha \cdot t_1 = C_2$, and finally for velocity projection on the axis «*x*» after integrating of the relation (3.52) over the variable we obtain

$$v_x(t) = a \cdot \cos\alpha \cdot t, \text{ if } t \le t_1;$$
$$v_x(t) = a \cdot \cos\alpha \cdot t_1, \text{ if } t \ge t_1. \quad (3.53)$$

This result is pre-clear because after the object dropping out from the airplane it has no the acceleration horizontal projection.

Let us research the vertical projection of the object velocity. Using the same considerations that were given when the equations (3.49) had been solved for derivatives we write

$$\frac{d^2 y}{dt^2} = \frac{dv_y}{dt} = a \cdot \sin\alpha, \text{ if } t \le t_1;$$
$$\frac{d^2 y}{dt^2} = \frac{dv_y}{dt} = -g, \text{ if } t \ge t_1. \quad (3.54)$$

From the beginning we will consider the case $t \le t_1$. As a result of solving of the first of the equations (3.54) we will obtain

$$v_y(t) = \frac{dy}{dt} = a \cdot \sin\alpha \cdot t + C_3,$$

where C_3 is an integration constant is determined from the condition $v_y(0) = C_3 = 0$ and therefore

$$v_y(t) = a \cdot \sin\alpha \cdot t, \text{ if } t \le t_1. \quad (3.55)$$

Further we will consider the case $t \ge t_1$. This case includes two of time intervals: the first of them is the interval $t_1 \le t \le t_c$ (t_c is time moment when the object occurs in the highest point of its trajectory after the object dropping out from the airplane), and the second interval $t \ge t_c$ is the time of the object falling on the ground after culmination by them the highest point of its trajectory. For this case we will use the second equality from (3.54), and thereafter we have

$$v_y(t) = -g \cdot t + C_4.$$

The integration constant C_4 is determined from the condition $v_y(t_1) = -g \cdot t_1 + C_4 = a \cdot \sin\alpha \cdot t_1$, whence $C_4 = a \cdot \sin\alpha \cdot t_1 + g \cdot t_1$, and then for the time points which belong to the interval $t_1 \le t \le t_c$ it may be written

$$v_y(t) = a \cdot \sin\alpha \cdot t_1 - g \cdot (t - t_1). \quad (3.56)$$

Finally from the equalities (3.55) and (3.56) for vertical projection of the object velocity vector we obtain

$$v_y(t) = a \cdot \sin\alpha \cdot t, \text{ if } t \le t_1;$$
$$v_y(t) = a \cdot \sin\alpha \cdot t_1 - g \cdot (t - t_1), \text{ if } t_1 \le t \le t_c. \quad (3.57)$$

The time moment in which object achieves the highest point of its trajectory is determined from the condition

$$v_y(t_c) = a \cdot \sin\alpha \cdot t_1 - g \cdot (t_c - t_1) = 0$$

whence it follows

$$t_c = \left(1 + \frac{a}{g} \cdot \sin\alpha\right) \cdot t_1 . \qquad (3.58)$$

Besides we will establish the law of changing the quantity $v_y(t)$ for the space of time $t \geq t_c$. Using the second of the equalities (3.54) we obtain

$$v_y(t) = -g \cdot t + C_5;$$

here C_5 is the next integration constant, for the determining of which the condition

$$v_y(t_c) = 0$$

has been used. This condition gives $C_5 = 0$ and as the result we have the next object velocity vertical projection changing law in the total time interval which is interested for us:

$$v_y(t) = a \cdot \sin\alpha \cdot t, \text{ if } t \leq t_1;$$
$$v_y(t) = a \cdot \sin\alpha \cdot t_1 - g \cdot (t - t_1), \text{ if } t_1 \leq t \leq t_c; \qquad (3.59)$$
$$v_y(t) = g \cdot (t_c - t), \text{ if } t_c \leq t.$$

At last we will find the object falling time on the ground from the highest point of its trajectory. For this purpose we will determine the coordinate « $y_{\max}$ » of the highest point of the object trajectory. In connection with mentioned above we have

$$y_{\max} = \int_0^{t_c} v_y(t) \cdot dt = y_1 + y_2 , \qquad (3.60)$$

where

$$y_1 = \int_0^{t_1} v(t) \cdot dt = \int_0^{t_1} a \cdot \sin\alpha \cdot t \cdot dt = \frac{1}{2} a \cdot \sin\alpha \cdot t_1^2;$$
$$y_2 = \int_{t_1}^{t_c} \left(a \cdot \sin\alpha \cdot t_1 - g \cdot (t - t_1)\right) \cdot dt = \qquad (3.61)$$
$$= \frac{1}{2} a \cdot \sin\alpha \cdot t_1^2 + a \cdot \sin\alpha \cdot t_1 \cdot t \cdot (t_c - t_1) - \frac{1}{2} g \cdot (t_c - t_1)^2;$$

here y_1 is the coordinate of the highest point of the object trajectory achieving by them when it was climbing together with the airplane, and y_2 is the addition to the coordinate mentioned above in consequence of the climb continuation of the object after its dropping out from the airplane. After inserting of the value time point t_c (formulae (3.54)) in the second of the equalities (3.61) and using the relation (3.56) we obtain

$$y_{\max} = \frac{1}{2} \cdot \left(1 + \frac{a}{g} \cdot \sin\alpha\right) \cdot a \cdot \sin\alpha \cdot t_1^2 . \qquad (3.61)$$

From the point with the coordinate $y_{\max}$ the object will make the free falling on the ground. The time point of this falling t_f enters in the expression

$$\int_{y_{\max}}^{0} dy = \int_{t_c}^{t_f} v(t)\cdot dt. \tag{3.62}$$

By using the third relation from given in (3.59) and inserting it into the previous equality (3.62) we have

$$y_{\max} = \frac{1}{2}\cdot g\left(t_f - t_c\right)^2. \tag{3.63}$$

If one will equate left parts of the expressions (3.60) and (3.63) and will solve the obtained equation in respect to t_f he can write

$$t_f = \left(1 + \frac{a}{g}\cdot \sin\alpha + \sqrt{\left(1 + \frac{a}{g}\cdot \sin\alpha\right)\cdot \frac{a}{g}\cdot \sin\alpha}\right)\cdot t_1. \tag{3.64}$$

For finding the total falling time which is interested for us we write

$$\tau = \tau_c + \tau_f, \tag{3.65}$$

where

$$\tau_c = t_c - t_1 \tag{3.66}$$

is the time climb of the object till the highest point of its trajectory after its dropping out from the airplane, and

$$\tau_f = t_f - t_c \tag{3.67}$$

is the time of its free falling from the point mentioned above. By using the values of t_c, t_f, determined by the relations (3.58) and (3.64) and the value of t_1, given in the problem condition and with the help of the relations (3.65) – (3.65) we find τ_c and τ_f. Then by inserting τ_c and τ_f values into the expression (3.65) we obtain the answer on the problem question:

$$\tau = \left(\frac{a}{g}\cdot \sin\alpha + \sqrt{\left(1 + \frac{a}{g}\cdot \sin\alpha\right)\cdot \frac{a}{g}\cdot \sin\alpha}\right)\cdot t_1. \tag{3.68}$$

Finally substituting numerical data given in the condition of the problem in the expression (3.68) we have

$$\tau = 102{,}0 sec.$$

2) We will ascertain on which of the part of its trajectory the object occurs in the point of time $t_3 = t_1 + t_2$. For this purpose we will calculate the moment of time t_c corresponds to the highest point of the object trajectory using the relation (3.58) and further we will compare it with the moment of time $t_3 = t_1 + t_2 = 6 sec$. For the moment of time t_c we have $t_c = 54{,}0 sec$. Thereby we see that in the case under consideration the inequality $t_c > t_3$ takes place and for the problem solving the second from the relations (3.53) and (3.57) must be used. Then using determination of the velocity modulus (3.6) we obtain

$$v(t_3) = \sqrt{v_x^2(t_3) + v_y^2(t_3)} = \sqrt{(a\cdot\cos\alpha\cdot t_1)^2 + (a\cdot\sin\alpha\cdot t_1 - g\cdot(t_3 - t_1))^2}. \tag{3.69}$$

Substitution of the numerical data given in the statement of a problem in the obtained formulae gives

$$v(t_3) = 970\, m/sec\,.$$

3) As the relation (3.36) shows for determining the object normal and tangential acceleration in fixed point of time it is necessary to know its velocity modulus as a function of time near the same point of time. With this purpose we will use the second relation from ones in (3.57) and the third relation from ones in (3.59) following which to determinate the object velocity modulus in the time interval defining by the third relation in (3.6) we have

$$v(t) = \sqrt{v_x^2(t) + v_y^2(t)} = \sqrt{(a \cdot \cos\alpha \cdot t_1)^2 + g^2 \cdot (t_c - t)^2}. \qquad (3.70)$$

For determining tangential acceleration we use the relations (3.19), (3.70) and as the result we will obtain .

$$a_\tau = \frac{dv}{dt} = \frac{g^2 \cdot (t_c - t)}{v} = g \cdot \frac{g \cdot (t_c - t)}{v} = g \cdot \frac{v_y}{v}. \qquad (3.71)$$

Concerning finding of the normal acceleration there are two ways for achieving of the formulated aim. The first of them supposes the strike using of the formula (3.36). This way occurs extremely tedious because it demands knowledge of the curve curvature radius in the point of the object falling. In one's term for determining this radius it is necessary to know the equation of the object trajectory what demands additional calculations.

Here there is the circumstance easing the problem solving. Indeed the object total acceleration modulus it is known to us and it is equal to $g = 9{,}8 m/sec^2$ if $t > t_1$. Then by using the expression (3.25) it is possible to determine the normal acceleration from the equality

$$a_n = \sqrt{g^2 - a_\tau^2}\,.$$

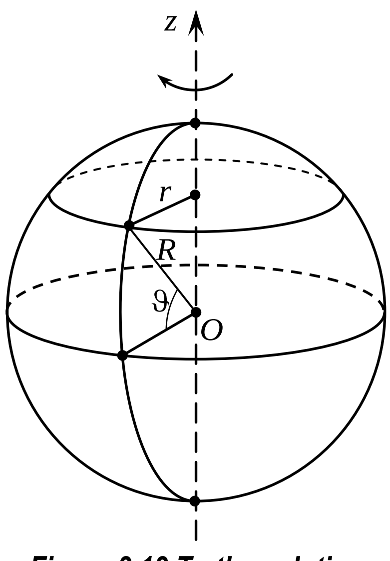

Figure 3.13 To the solution of the problem 3.2

By substituting into given equality the value a_τ determined from the relation (3.71) we have

$$a_n == g \cdot \frac{v_x}{v}. \qquad (3.72)$$

The expressions (3.71) and (3.72) are the solutions of the problem in the common case. Now we are proceeding to numerical calculations. In our case $t = t_f = \tau + t_1 = 106 sec$, consequence $a_\tau \to a_\tau(t_f) = g \cdot \frac{v_y(t_f)}{v(t_f)} = -7{,}5 m/sec^2$. The negative sign in the obtained value is connected with the negativity of the vector velocity projection on the axis «*y*» (figure (3.13));. By analogy with the tangential acceleration for normal projection we obtain $a_n(t_f) = g \cdot \frac{v_x(t_f)}{v(t_f)} = 6{,}3 m/sec^2$.

2. For the point of the earth's surface located at a Kiev latitude ($\vartheta = 50^{\circ}27'$) determine the linear velocity v of its daily rotation and also its normal acceleration (for Earth's radius the value $R = 6{,}4 \cdot 10^6 m$ may be taken).

<u>Solution.</u> From figure 3.13 we see that the point mentioned above rotates along the circle with radius $r = R \cdot \cos\vartheta$ having the angular velocity coincident with ones of the daily Earth rotation. This quantity is equal to $\omega = 2\pi/T$, $T = 8{,}64 \cdot 10^4 sec$ is a period of the mentioned rotation. Using the connection between the linear and angular velocities (3.43) and making required substitutions finally we obtain

$$v = \omega \cdot r = (2\pi \cdot R/T) \cdot \cos\vartheta = 296{,}4\, m/sec.$$

3. The particle moves along the axis «x» and its velocity projection changes in according to the law $v_x(t) = A + Bt, A = 4\, sm/sec, B = -2 sm/sec^2$ beginning motion from the origin (the point $x(0) = 0$). Determine: 1) the particle coordinates $x(t_i)$ in points of time $t_1 = 2sec, t_2 = 4sec, t_3 = 6sec$, measured from the time origin; 2) path S traversed by the particle over time interval during the time interval from the time origin till the point of time $t_3 = 6sec$.

<u>Solution. 1)</u> Using the formula (3.27) for coordinate «x» of particle we write

$$x(t_i) = x(0) + \int_0^{t_i} v_x(t) \cdot dt \tag{3.73}$$

where symbol «i» runs the values $i = 1,2,3$. Inserting the expression for $v_x(t)$ given in the problem condition into (3.73) and taken into account that $x(0) = 0$ we have

$$x(t_i) = \int_0^{t_i} (A + Bt) \cdot dt = At_i + \frac{Bt_i^2}{2}, \tag{3.74}$$

what is the answer on the question of the problem in the common case. For various meanings «i» by using of the expression (3.74) and the numerical data given in the problem condition we obtain

$$x(t_1) = At_1 + \frac{Bt_1^2}{2} = 4sm;\ x(t_2) = At_2 + \frac{Bt_2^2}{2} = 0;\ x(t_3) = At_3 + \frac{Bt_3^2}{2} = 12sm.$$

2) Concerning calculating the path traversed by the particle we will proceed from the expression (3.12) whence it follows that to answer the question problem we must have the law of particle velocity modulus changing in respect to time. Using the problem condition we write

$$S = \int_0^{t_3} |A + Bt| \cdot dt = \int_0^{t'} |A + Bt| \cdot dt + \int_{t'}^{t_3} |A + Bt| \cdot dt. \tag{3.75}$$

In given expression $t' = 2s$ and it is the root of the equation $A + Bt' = 0$. Making corresponding substitutions to the equality (3.75) we obtain

$$S = 20sm.$$

4. The particle emitted by the source passes the distance L with constant speed. Then it begins to brake moving with acceleration a. Determine the speed v_s under which the total time of the particle motion is the smallest.

Solution. The total time of the particle motion is equal to

$$t = t_1 + t_2, \tag{3.76}$$

where $t_1 = \dfrac{L}{v}$ is the time of its uniformly motion with constant velocity, and $t_2 = \dfrac{v}{a}$ is the particle motion time after the braking appearing. Then the equality (3.69) may be written in the form

$$t = \frac{L}{v} + \frac{v}{a} = \left(\sqrt{\frac{L}{v}} - \sqrt{\frac{v}{a}}\right)^2 + 2\sqrt{\frac{L}{a}}. \tag{3.77}$$

As it seen from the relation (3.77) time t is the smallest if $\left(\sqrt{\dfrac{L}{v}} - \sqrt{\dfrac{v}{a}}\right)^2 = 0$, or if

$$v_s = \sqrt{La}.$$

5. In the process of the particle motion along the planar curve radius-vector of the particle changes in according to law $\vec{r}(t) = A\vec{i} \cdot \sin\omega t + B\vec{j} \cdot \cos\omega t$, where $A = 2sm, B = 3sm$. Determine: 1) the equation of the particle trajectory in the manifest form; 2) particle velocity modulus as the functions of the particle coordinate «x»; 3) the largest and the smallest values of the particle velocity modulus as the functions of the particle coordinate «x»; 4) the total, tangential and normal accelerations of the particle as the time functions; 5) the total, tangential and normal accelerations of the particle as functions of the particle coordinate «x»; 6) the largest and the smallest values of the total, tangential and normal accelerations of the particle as functions of the particle coordinate «x»;

Solution. 1) The vector equation given in the text of the problem is equivalent to following two scalar equations:

$$\begin{aligned} x(t) &= A\sin\omega t; \\ y(t) &= B\cos\omega t. \end{aligned} \tag{3.78}$$

Set of the equations (3.78) represents the ellipse equation in the parametrical form. For obtaining the ellipse equation in the manifest form we write $x(t)/A = \sin\omega t;\ y(t)/B = \cos\omega t$ whereupon we obtain

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1, \tag{3.79}$$

and using the problem data for the equation in the manifest form finally we have

$$\frac{x^2}{4} + \frac{y^2}{9} = 1.$$

2) Using the relation (3.78) we find the velocity vector projections:

$$v_x(t) = \frac{dx}{dt} = \omega A \cdot \cos\omega t;$$
$$v_y(t) = \frac{dy}{dt} = -\omega B \cdot \sin\omega t. \qquad (3.80)$$

With the help of the relations (3.6), (3.80) we write

$$v(t) = \sqrt{v_x^2(t) + v_y^2(t)} = \omega\sqrt{A^2\cos^2\omega t + B^2\sin^2\omega t}, \qquad (3.81)$$

and with the help of the relation (3.79) we may represent the particle coordinate «y» as a function of its coordinate «x»:

$$y = \pm B\sqrt{1 - \frac{x^2}{A^2}} \qquad (3.82)$$

then for the answer on the question of the problem by using relations (3.79) and (3.75) we obtain

$$v(x) = \omega\sqrt{A^2 + \left(B^2/A^2 - 1\right)\cdot x^2}. \qquad (3.83)$$

3) The largest and the smallest values of the particle velocity modulus as the functions of the particle coordinate «x» is determined by the analysis given near.

Let us consider the cases $A > B$ and $A < B$ separately.

a) $A > B$. Using (3.76) we have

$$v_{larg} = \omega A, \text{ if } x = 0; v_{small} = \omega B, \text{ if } x = \pm A;$$

b) $A < B$. Repeating previous reasoning we obtain:

$$v_{larg} = \omega B, \text{ if } x = \pm A; v_{small} = \omega A, \text{ if } x = 0.$$

4) From the equalities (3.17) and (3.80) we determine the total acceleration as a time function:

$$a(t) = \sqrt{a_x^2(t) + a_y^2(t)} = \sqrt{\left(\frac{dv_x}{dt}\right)^2 + \left(\frac{dv_y}{dt}\right)^2} =$$
$$= \omega^2 \cdot \sqrt{A^2\sin^2\omega t + B^2\cos^2\omega t}. \qquad (3.84)$$

For the tangential acceleration by using the relations (3.20), (3.81) we have

$$a_\tau(t) = \frac{dv}{dt} = \left|\frac{\vec{a}\cdot\vec{v}}{v}\right| = \frac{\omega^2}{2}\cdot\frac{\left|\left(B^2 - A^2\right)\cdot\sin 2\omega t\right|}{\sqrt{A^2\cos^2\omega t + B^2\sin^2\omega t}}. \qquad (3.85)$$

We find the expression for the normal acceleration with the help of the relations (3.25), (3.85):

$$a_n(t) = \sqrt{a^2 - a_\tau^2} = \left|\frac{a_x\cdot v_y - a_y\cdot v_x}{v}\right| = \frac{\omega^2\cdot AB}{\sqrt{A^2\cos^2\omega t + B^2\sin^2\omega t}}. \qquad (3.86)$$

5) For representation of the expressions for the a, a_τ and a_n as functions of the point coordinate «x» at the curve we will use the equalities (3.82), (3.84) – (3.86). Then we finally obtain

$$a=\omega^2\cdot\sqrt{x^2+y^2}=\omega^2\cdot\sqrt{B^2+\left(1-B^2/A^2\right)x^2};$$

$$a_\tau=\omega^{2\cdot}\cdot A\cdot\frac{\left|\left(B^2/A^2-1\right)\cdot xy\right|}{AB\cdot\sqrt{1+\left(B^2/A^2-1\right)\cdot\left(x/A\right)^2}}=$$

$$=\omega^{2\cdot}\cdot A\cdot\left|\left(B^2/A^2-1\right)\right|\sqrt{\frac{\left(x/A\right)^2-\left(x/A\right)^4}{1+\left(B^2/A^2-1\right)\cdot\left(x/A\right)^2}};\tag{3.87}$$

$$a_n=\frac{\omega^{2\cdot}\cdot B}{\sqrt{1+\left(B^2/A^2-1\right)\cdot\left(x/A\right)^2}}.$$

6) In order to obtain the answer on the question formulated in the problem condition we will consider the cases $A>B$ and $A<B$ separately.

a) $A>B$. Using (3.80) we have

$$a_{larg}=\omega^2A,\text{if }\ x=\pm A;\ a_{small}=\omega^2B,\text{if }\ x=0;$$

$$a_{n,larg}=\omega^2A,\text{if }\ x=\pm A;\ a_{n,small}=\omega^2B,\text{if }\ x=0;$$

b) $A<B$. Repeating previous reasoning we obtain:

$$a_{larg}=\omega^2B,\text{if }\ x=0;\ a_{small}=\omega^2A,\text{if }\ x=\pm A;$$

$$a_{n,great}=\omega^2B,\text{if }\ x=0;\ a_{n,small}=\omega^2A,\text{if }\ x=\pm A.$$

Finding the largest and the smallest values of the tangential acceleration is more complicated problem. For its solving we will to proceed from the expression (3.80) for the tangential acceleration. Let us introduce the variable $\xi=\left(x/A\right)^2$ and consider the function

$$f\left(\xi\right)=\frac{\xi-\xi^2}{1+c\xi},\tag{3.88}$$

where $c=B^2/A^2-1$. The function (3.88) enters in the formula (3.87) for the tangential acceleration of the particle. Analysis of this function is sufficient to receive the answer on the question which is interesting for us. The function (3.88) vanishes if $\xi=0$ and if $\xi=1$, and must obey the conditions

$$\begin{gathered}f\left(\xi\right)\geq 0,\\ \left|\xi\right|\leq 1\end{gathered}\tag{3.89}$$

(see formula (3.80) for tangential acceleration) and thus at the points $\xi=0;1$ it reaches the smallest value. Then the function (3.88) gets the greatest value at the point which is the root of the equation $f'\left(\xi\right)=0$. As it follows from (3.88) the noted equation has the form

$$f'\left(\xi\right)=c\xi^2+2\xi-1=0.$$

The roots of this equation are such:

$$\xi_1 = \frac{\sqrt{c+1}-1}{c}, \xi_2 = -\frac{\sqrt{c+1}+1}{c} \tag{3.90}$$

The first of these roots (3.90)

$$\xi_1 = \frac{A}{A+B} \tag{3.91}$$

satisfies both conditions (3.89) but the second of them

$$\xi_2 = \frac{A}{A-B}$$

does not satisfy requirements (3.89) and therefore should be rejected. Thereby the smallest value of the tangential acceleration, which is equal to zero, it accepts at the points

$$x = 0; x = \pm A,$$

which are the points of intersection of the particle trajectory with the coordinate axes. As it is seen from the equality (3.91) and follows from the determination of variable ξ the largest value of the tangential acceleration it reaches at the points

$$x = A \cdot \sqrt{\frac{A}{A+B}}; x = -A \cdot \sqrt{\frac{A}{A+B}} \ . \tag{3.92}$$

By substitution of the quantities (3.92) into (3.87) for the largest value of the tangential acceleration finally we have

$$a_\tau = \omega^2 |A-B|.$$

6. The projection of the disc angular velocity having the fixed rotation axis changes in according to law $\omega_z(t) = A + Bt$, $A = 15\,rad/sec$, $B = -7{,}5\,rad/sec^2$. Determine the number N of rotations made by the disc from the point of time $t = 0$ to its total stopping.

Solution. First of all we will determine the time passed from the beginning of the disc rotation until its total stopping. This time τ is determined from the condition

$$\omega_z(\tau) = A + B\tau = 0, \tag{3.93}$$

whence it follows $\tau = -A/B = 2s$. Thereafter we will find the rotation angle of the disc during the time interval found above. Using the relation (3.93) as a result we have

$$\varphi = \int_0^\tau \omega_z(t) \cdot dt = \int_0^\tau (A + Bt) \cdot dt = A\tau + \frac{1}{2} B\tau^2 = 15\,rad.$$

The number N of the rotations made by the disc from the point of time $t = 0$ to its total stopping will be determined from the relation $N = \varphi/2\pi$ what finally give

$$N = \frac{15}{2\pi} = 2{,}39.$$

Control task

3.1. Analyse how many frames of reference can be linked to one body of reference: a) one; b) three; c) the number frames of reference coincides with the number of axes of symmetry of the body reference frame; d) infinite.

3.2. Analyse how many frames of reference can be linked with a fixed point on a given body of reference: a) one; b) three; c) infinite; d) the number frames of reference coincides with the number of axes of symmetry of the body reference

3.3. Analyse how many coordinate systems can be linked to one body of reference: a) one, b) infinite; c) the number of coordinate systems coincides with the number of axes of symmetry of the body reference frame; d) three.

3.4. Analyse how many coordinate systems can be linked with a fixed point on a given body of reference: a) one; b) three; c) infinite; d) the number of coordinates coincides with the number of axes of symmetry of the body of reference.

3.5. Investigate the properties inherent in derivative of path by time: a) it is positive and it is continuous; b) it is positive and it is differentiable; c) it is negative and it is non-differentiable; d) it has an arbitrary sign and it is continuous.

3.6. Analyse which of the following relations for the time derivatives of path takes place: a) $\frac{dS}{dt} \geq 0$; b) $\frac{dS}{dt} < 0$; c) $\frac{dS}{dt} \geq c$; d). $\frac{dS}{dt} \leq 0$.

3.7. Analyse whether the modulus of the particle displacement vector over certain time may be equal to the path passed by the particle over the same time: a) it is always equal; b) it is equal in the case of rectilinear motion of the particle without changing its direction; c) it is equal in the case of rectilinear motion of the particle with a changing of motion direction at the some time points; d) equality never hold.

3.8. Investigate which of the following relationships for the increment of the particle radius - vector modulus $d|\vec{r}| = dr$ and for the increment of its displacement vector modulus $|d\vec{r}|$ it takes place: a) $dr = |d\vec{r}|$; б) $dr \leq |d\vec{r}|$; в) $dr \geq |d\vec{r}|$; г) $dr > |d\vec{r}|$.

3.9. Analyse whether the equality $dr = |d\vec{r}|$ ($\vec{r}$ – radius vector) is fulfilled and, if so, point out which of these cases is the case: a) it is fulfilled for the rectilinear motion of a particle; b) it has not never fulfilled; c) it is fulfilled for the particle motion along an arbitrary closed trajectory , d) it is fulfilled for the motion of a particle along the circle.

3.10. Investigate which of the following expressions determines the modulus of the particle average velocity, and which of them determines the modulus average value: a) $\left\langle \frac{d|\vec{r}|}{dt} \right\rangle$ is modulus of the average velocity $\left|\left\langle \frac{d\vec{r}}{dt} \right\rangle\right|$ is the average value of the velocity modulus; b) $\left|\left\langle \frac{d\vec{r}}{dt} \right\rangle\right|$ is the average velocity modulus, $\left\langle \left|\frac{d\vec{r}}{dt}\right| \right\rangle$ is the average value of the velocity modulus; c) $\left\langle \frac{d|\vec{r}|}{dt} \right\rangle$ is the average velocity modulus, $\left| \frac{d\langle \vec{r} \rangle}{dt} \right|$ is the velocity modulus average value; d) $\left| \frac{d\langle \vec{r} \rangle}{dt} \right|$ is the modulus of the average velocity, $\left\langle \left| \frac{d\vec{r}}{dt} \right| \right\rangle$ is the average value of the velocity modulus. .

3.11. Particle, which performs rectilinear motion in a time changes direction of motion. To analyze which of the following quantities changes sign at specified time: a) the projection of the radius – vector; b) the projection of the displacement vector; c) projection of the velocity vector; d) there is no such value.

3.12. Investigate the motion of the particle moving along the axis *OX* in according to law $x(t) = A + Bt + Ct^2$, where *A* = -19*m*, *B* = 20*м/sec*, *C* = -1*m/sec*^{2}, and find the projection of its acceleration a_x on the axis *OX*: a) a_x = 2*m/sec*^{2}; b) a_x = - 2*m/sec*^{2}; c) a_x = - 20*m/sec*^{2}; d) a_x = 20 *m/sec*^{2}.

3.13. The motion of the particle is given by the equation $x = At + Bt^2$,

A=0,25*m/sec*, *B* = - 0,5*m/sec*^{2}. Investigate its motion for the first *t* = 0,25*sec*, and determine the path *S*, which the particle passes during this period: a) *S*= 3,125·10^{-2}*m*; b) *S*= 3,125·10^{-3}*m*; c) *S*= 3,125·10^{-4}*m;* d) *S*= 0,3125*m.*

3.14. The motion of the particle takes place in according to law $x = A + Bt^2 + Ct^3$, where, B = -3*m/sec*^{2}, C = 2*m/sec*^{3}. Exploring its motion establish the average value of particle velocity modulus during the time interval from the point of time $t_1 = 0{,}5sec$ till the point $t_1 = 1{,}5sec$ that has passed since the motion beginning: a) $\langle v \rangle$ = 2*m/sec*; b) $\langle v \rangle$ = 2,5*m/sec*; c) $\langle v \rangle$ = 1,5*m/sec*; d) $\langle v \rangle$ = 1*m/sec.*

3.15. Investigating the body free falling from a fixed height, compare its average velocity $\langle v \rangle_1$ during the flight to a point that lies in its midway, with the average velocity $\langle v \rangle_2$ during the falling time, and average velocity $\langle v \rangle_3$ for the time which is equal to half of the falling time, and specify the correct ratios: a) $\langle v \rangle_1$ < $\langle v \rangle_2$ < $\langle v \rangle_3$; b) $\langle v \rangle_1$ < $\langle v \rangle_3$ < $\langle v \rangle_2$; c) $\langle v \rangle_2$ < $\langle v \rangle_1$ < $\langle v \rangle_3$; d) $\langle v \rangle_3$ < $\langle v \rangle_1$ < $\langle v \rangle_2$.

3.16. Analyse the conditions under which the path of a particle moving along the axis *Ox*, defined by the formula $S = v_0 t + \frac{a_x t^2}{2}$, where v_0 is an initial velocity, a_x is an acceleration projection, *t* is the moving time: a) provided $a_x > 0$, and provided $a_x < 0$ only till a stopping; b) only provided $a_x > 0$; c) for any conditions; d) the correct answer is absent.

3.17. As a result of the analysis point out which from the components of the acceleration vector are converted into zero in the following cases: a) both normal and tangential acceleration are converted into zero during uniform motion along a circle; b) normal acceleration is converted into zero during uniform motion along a circle but tangential acceleration is converted into zero while rectilinear uniform motion; c) tangential acceleration is converted into zero while rectilinear accelerated motion; d) tangential acceleration is converted into zero during uniform motion along a circle and normal acceleration is converted into zero while the particle executes the accelerated rectilinear motion.

3.18. As a result of the analysis point out the correct relationship between the modules of the normal component of the acceleration vector a_n, of its tangential component a_τ and of its total acceleration modulus a: a) $a_n \approx a_t$, $a_n > a$; b) $a_n < a_t$, $a_n \geq a$; c) $a_n = a_t \approx a$; d) $a_n \leq a$, $a_t \leq a$.

3.19. As a result of the analysis point out the correct relationship between the velocity vector and the normal acceleration, tangential acceleration, and total acceleration of the particle that executes a various motion along an arbitrary curvlinear trajectory: a) $\vec{v}\,\vec{a}_n = 0$, $\vec{a}_n\,\vec{a}_t = 0$; b) $\vec{v}\,\vec{a}_t = 0$, $\vec{a}_t\,\vec{a}_n > 0$; c) $\vec{v}\,\vec{a} = 0$, $\vec{a}_t\,\vec{a}_n = a_t a_n$; d) $\left[\vec{v}\vec{a}\right] = 0$, $\left|\left[\vec{a}_t\vec{a}_n\right]\right| = a_t a_n$.

3.20. The particle moves along the circle so that its velocity modulus increases with the time. Analyse (see the figure 3.14) to which of the cases this motion corresponds if $\vec{a}$ is a total acceleration vector of the particle:

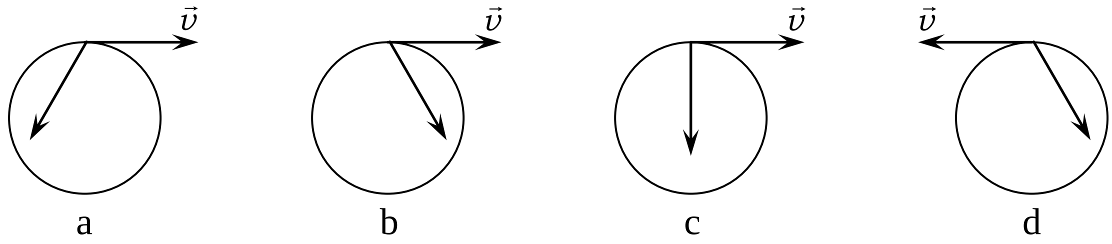

Figure 3.14. To the control task 3.20

3.21. The particle radius-vector depends on time according to the law $\vec{r} = At^2\vec{i} + Bt\vec{j}$ where $A = 9 m/sec^2$, $B = 6 m/sec$. Investigate the motion of a particle and give the equation of particle trajectory in the explicit form: a) $y = 2\sqrt{x}$; b) $y = \sqrt{x}$; c) $y = \sqrt{x/2}$; d) $y^2 = 4x$.

3.22. Investigate the motion of a particle, for which the motion law has the form $\vec{r} = At\vec{i} + Bt^2\vec{j}$, where $A = 2\, m/sec$, $B = 3\, m/sec^2$, and determine its velocity $\vec{v}$ at the moment of time $t = 1 sec$: a) $\vec{v} = A\vec{i} + 2B\vec{j}$, *m/sec*; b) $\vec{v} = A\vec{i} + B\vec{j}$, *m/sec*, c) $\vec{v} = A\vec{i} + B\vec{j}$, *m/sec*, d) $\vec{v} = 2B\vec{j}$, *m/sec.*

3.23. Particle radius-vector changes in time according to the law $\vec{r}(t) = \left(A + Bt + Ct^2\right)\vec{i} + \left(Dt + E\right)\vec{j} + F\vec{k}$, where $B = -4\, m/sec$, $C = 2\, m/sec^2$, $D = 6\, m/sec$. Investigating the particle motion, determine module of its tangential a_τ and normal a_n acceleration in point of time $t = 3 sec$ that passed from the beginning of the motion and the trajectory curvature radius R, in which the particle turns out in the specified point of time:

a) $a_\tau = 10\,m/sec^2$, $a_n = 3{,}6\,m/sec^2$, $R = 26{,}4m$;

b) $a_\tau = 5{,}1\,m/sec^2$, $a_n = 3{,}2\,m/sec^2$, $R = 22{,}4m$;

c) $a_\tau = 3{,}2\,m/sec^2$, $a_n = 2{,}4\,m/sec^2$, $R = 41{,}7m$;

d) $a_\tau = 2{,}4\,m/sec^2$, $a_n = 3{,}2\,m/sec^2$, $R = 16{,}6m$;

3.24. Investigate the correlation between the vectors of the linear and angular particle velocities, which rotates along a circle, and specify true from its: a) $[\vec{\omega}\vec{v}] = 0$; b) $\left|[\vec{\omega}\vec{v}]\right| = \omega v$; c) $\vec{\omega}\vec{v} = 0$; d) $\vec{\omega}\vec{v} = \omega v$.

3.25. Analyse whether the projection of linear and angular velocity vectors on the coordinate axes are changing when the reflection of the coordinate axes takes place: a) the projection of the linear velocity is changing its sign on the opposite one, and the angular velocity does not ; b) the projection of the angular velocity is changing its sign on the opposite and the projection of the linear one does not; c) the projections of both velocities do not changing their signs; d) the projections of both velocities are changing their signs on the opposite.

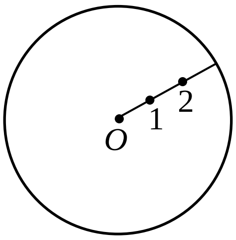

Figure 3.15.
To the control task 3.26

3.26. Points 1 and 2 (see figure 3.15) lie on the same radius of the rotating shaft. Analyse which of the noted quantities describing the movement are the same for these points: a) the angular velocity and linear acceleration; b) the angular and linear velocities; c) angular velocity and angular acceleration; d) linear velocity and angular acceleration.

3.27. The rotation angle of the body rotating about an axis varies according to the law $\varphi = At^4 + Bt^2 + C$, where $A = 0{,}5 rad/sec^4$, $B = 1 rad/sec^2$. Determine the total acceleration of the body point, which is located at a distance $R = 0{,}4m$ from the axis of rotation: a) $a = 5{,}24\,m/sec^2$; b) $a = 6{,}47\,m/sec^2$; c) $a = 15{,}63\,m/sec^2$; d) $a = 13{,}29\,m/sec^2$.

3.28. Considering electron in the electrical field of atomic nucleus and assuming both as material points indicate the number of freedom degrees of electron in this system: a) one; b) two; c) three; d) four.