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ELEMENTARY PARTICLE PHYSICS AND FIELD THEORY

ON THE CHARGE FORM FACTOR AND EIGENVALUE OF FERMION ENERGY IN A THREE-DIMENSIONAL ELECTRODYNAMICS

M. Sh. Pevzner

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A static field and self-energy of a particle are considered for a particle charge distributed throughout a 2 + 1-measurement space. The potential of the static field for $r \to \infty$ has the same asymptotics as for the delta form factor, provided an account is taken of the contribution from vacuum polarization; at the origin of coordinates, the above potential is regular. The proposed form factor allows a relation for the particle charge distribution to be derived in a closed form. The diagonal tension-tensor components of the particle-generated field are found to vanish and the particle field mass calculated using the classical method appears to be finite in the case where the proposed form factor is used. This mass coincides with that obtained through quantum calculations by the order of magnitude.

1. Recently various aspects of electrodynamics in the 2 + 1 space have attracted numerous researchers. This interest is due to the fact that there are certain hopes to gain a deeper insight into a number of macroscopic quantum effects such as high-temperature superconductivity and quantum Hall effect [1]. In this model, as in a four-dimensional case, there occurs a dynamic breaking of chiral symmetry, which leads, in particular, to the appearance of a dynamic particle mass [2–4].

A hypothesis on the pure field origin of the particle mass in a four-dimensional electrodynamics is confronted by a serious handicap due to the fact that the field energy and momentum of a charged particle at rest form no Lorentz vector [5]. It was also considered that the statement of the problem on the mechanism of particle emergence makes no sense in classical physics since the result obtained is not a limiting case in the quantum system for $\hbar \to 0$. However, it was shown in [6–8] that a more accurate calculation of radiative corrections to the particle mass allows the limiting transition to be performed in such a way that the correspondence principle between quantum and classical physics is observed. Therefore, we believe that it is worthwhile to consider the problem from a classical physical standpoint first and then study it in the quantum formulation. As for the problem concerned with the fact that the particle field energy and momentum form no Lorentz vecor, it is due to the presence of nonelectric forces inside the particle, which is manifested in nondiagonal tension-tensor components of the particle field being different from zero [5]. This problem, however, seems not to arise in a three-dimensional electrodynamics when considering vacuum loops with massless fermions [9, 10]. In this case, it is suggested that the classical integral of particle energy is truncated at small distances corresponding to the electric radius of the particle, which, in turn, is equivalent to a hypothesis on the concentration of particle charge at its periphery.

This paper studies the static field and self-energy of a particle in the 2 + 1-measurement space. The particle charge is distributed over its volume. In the 3 + 1-measurement space there is an analogy between the dynamic breaking of chiral symmetry and the phenomenon of "fall onto the center" in a strong Coulomb field in a single-particle problem considered within the relativistic quantum mechanics [11]. This circumstance lends impetus to considering an analogous problem for the case of the 2 + 1-measurement space. The "fall onto the center" in a strong Coulomb field in the 2 + 1-measurement space is discussed using a two-dimensional representation of the Clifford algebra [12], which, however, has no direct relationship to the problem under study, since chiral symmetry is absent and the mass term breaks spatial and time parity in the case of the two-dimensional representation [13]. On the other hand, the point-charge field in QED_3 differs from the Coulomb one even in the absence of confinement [2]. In the presence of confinement, the structure of this field is even

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more complicated [14]. Therefore there are the following ways of solving the corresponding quantum-mechanical problem: a) numerical solution of the Dirac equation in the 2 + 1-measurement space with the exact potential for the charge-point field and b) approximation of the potential by the functions admitting an analytical solution of the Dirac equation in the 2 + 1-measurement space, which can turn out to be equivalent to an assumption of a certain charge distribution within the particle. Giving no consideration to the first possibility, we note that in the presence of confinement these approximations are discussed in [14, 15]. For a number of reasons considered in [9, 10], of interest is the case where confinement is absent. It is this case that is discussed in the paper.

2. From the relation for the charge static-field potential, we get

$$A_0(\boldsymbol{r}) = \frac{i}{(2\pi)^3} \int G_R(\boldsymbol{k},0)\ \varphi(\boldsymbol{k})\ e^{i\boldsymbol{kr}} d^2\boldsymbol{k}\,, \tag{1}$$

where $G_R(\boldsymbol{k},0)$ is the regularized transverse part of a net photon propagator at zero frequency and $\varphi(\boldsymbol{k})$ is the charge form factor of the particle determined by the relation

$$\varphi(\boldsymbol{k}) = \int Q(\boldsymbol{r}') e^{i\boldsymbol{kr}} d^2\boldsymbol{k}\,. \tag{2}$$

Here the function $Q(\boldsymbol{r})$ characterizes the particle charge distribution and satisfies the following condition:

$$Q = \int Q(\boldsymbol{r}') d^2\boldsymbol{r}'\,, \tag{3}$$

where Q is the total particle charge on the plane. Using the integral representation for the zeroth-order Bessel function and allowing for axial symmetry of the problem, we can write

$$\varphi(k) = 2\pi \int_0^\infty Q(r') J_0(kr') r'dr'\,. \tag{4}$$

Let us return to considering integral (1). Performing integration over the angle, we get

$$A_0(r) = \frac{i}{2\pi} \int G_R(k,0)\varphi(k) J_0(kr) kdk\,. \tag{5}$$

The function $G_R(k,0) = G_R(k)$ in Eqs. (2) and (5) is determined by the relation

$$G_R(k) = \frac{1}{ik^2 - \Pi_R(k)}\,, \tag{6}$$

where $\Pi_R(k) = \frac{1}{2}\left(\Pi_{\mu\mu}(k) - \Pi_{\mu\mu}(0)\right)$ and $\Pi_{\mu\mu}(k)$ is the spur of a polarization operator to determine which we must solve a set of the Schwinger – Dyson equations from quantum electrodynamics for the case of two spatial and one temporal measurements (QED_3), which is of exceptional difficulty. Hence, certain approximations are used for solving the problem. We can restrict ourselves, for example, to the approximation used for solving the problem of dynamic breaking of chiral symmetry in QED_3 [7–9]. This is to apply the N^{-1}-expansion and to neglect mass function when calculating $\Pi_R(k)$, which gives [7]

$$\Pi_R(k) = -i\frac{\alpha k}{8}\,, \tag{7}$$

where $\alpha = e_0^2 N$, e_0^2 is the nonrenormalized coupling constant in QED$_3$ and N is the number of fermions in the model. Using Eqs. (5) – (7), we have

$$A_0(r) = \frac{1}{2\pi}\int_0^\infty \frac{\varphi(k) J_0(kr)}{k+\alpha/8} dk . \tag{8}$$

Nothing has been said, so far, concerning an explicit form of the form factor $\varphi(k)$. We require that at large distances $(\alpha r/8 >> 1)$ this form factor leads to the same potential as the delta one ($\varphi(k)=1$). On the other hand, it must satisfy condition (3). Needless to say that the form of form factor is not unambiguously limited by the foregoing requirements. As will be seen later, this circumstance is of no great importance for our problems.

If the form factor is chosen in the form

$$\varphi(k) = Q\frac{k+\alpha/8}{\alpha/8} e^{-\frac{k}{\alpha/8}} , \tag{9}$$

we get the following relation for the static field:

$$A_0(r) = \frac{Q}{2\pi\alpha/8}\int_0^\infty e^{-\frac{8k}{\alpha}} J_0(kr) dk = \frac{Q}{2\pi}\frac{1}{\left(1+(\alpha r/8)^2\right)^{1/2}} . \tag{10}$$

We derive the following relation for the field strength

$$E_r(r) = -\frac{\partial A_0}{\partial r} = \frac{Q}{2\pi r}\frac{(\alpha r/8)^2}{\left(1+(\alpha r/8)^2\right)^{3/2}} . \tag{11}$$

It is easily seen that the behavior of the values $A_0(r)$ and $E_r(r)$ is changed for small r as compared with the delta factor. As for the behavior of these values at large distances, it remains unchanged. This seems quite natural since the field does not “perceive” the charge distribution within the source.

3. Let us consider the problem of tension tensor in a three-dimensional electrodynamics. Nondiagonal components of tensor density are zero from symmetry considerations. As for the diagonal components, we have

$$T_{ii}^{(0)} = E_i^{02} - \frac{1}{2}E^2 \tag{12}$$

(summation over indices is not implied). The symbol $^{(0)}$ suggests that the corresponding value is taken in the rest frame of the particle; in addition, the system of units is used where $\hbar = c = 1$and $i = 1,2$.

For the particle mass to be of field origin, it is necessary that its energy and momentum form the Lorentz vector. This condition is met if the following relation holds:

$$\int T_{ii}^{(0)} d^2\boldsymbol{r}^{(0)} = 0 . \tag{13}$$

In our case this relation is fulfilled. Indeed,

$$\int T_{ii}^{(0)} d^2\boldsymbol{r}^{(0)} = \pm\frac{1}{2}\int_0^\infty E^{(0)2} r dr \int_0^{2\pi} \cos 2\alpha \, d\alpha . \tag{14}$$

Here the sign «+» corresponds to the component $T_{11}^{(0)}$, and the sign «–» – to the component $T_{22}^{(0)}$. The internal integral in the right side of Eq. (14) is zero. At the same time, representation of the double integral as an iterated integral is admissible only in the case where the integrals involved in the iterated integral converge. As is seen from Eq. (11) for $E_r\left(r\right)=E^{(0)}$, this is the case. Thus, one of the most serious difficulties for the development of a hypothesis on a field mass of a particle in classical physics associated with the fact that the diagonal components of the field-tension tensor density of the particle are nonzero, are overcome in a three-dimensional electrodynamics. This is due to using quantum representations (in Eq. (1), the Green's function including radiative corrections is used).

4. Let us calculate the energy and mass of a charged particle. We have

$$W_0=\int T_{44}^{(0)}d^2\boldsymbol{r}^{(0)}=\frac{1}{2}\int E_r^{(0)2}d^2\boldsymbol{r}^{(0)} . \tag{15}$$

Substituting Eq. (11) into Eq. (15), we get

$$W_0=m_0=\frac{1}{4\pi}Q^2 , \tag{16}$$

where W_0 and m_0 are the linear energy density and mass of the particle, respectively. For the corresponding total values, we have

$$W=m=\frac{1}{4\pi}{e_0}^2=\frac{1}{4\pi N}\alpha . \tag{17}$$

This result is remarkable due to the fact that it demonstrates proportionality between the particle mass and the dimensional constant α. The presence of this proportionality is obvious since e_0^2 (or α) is the only model constant with the required dimension. The fact that the choice of the form factor as Eq. (9) results in our case in a finite mass of the particle, whereas the choice of the delta form factor leads to the divergence of integral (15) at small distances and the necessity of sharp truncation of the corresponding integral [9, 10].

5. Let us find a spatial distribution of the particle charge. It follows from Eq. (2) that

$$Q\left(r\right)=\frac{1}{2\pi}\int_0^\infty k\varphi\left(k\right)J_0\left(kr\right)dk . \tag{18}$$

Substituting Eq. (9) into Eq. (18), we get

$$Q\left(r\right)=\frac{\left(\alpha/8\right)^2}{2\pi}Q\left(2F\left(3/2,2,1;-\left(\alpha r/8\right)^2\right)+F\left(1,3/2,1;-\left(\alpha r/8\right)^2\right)\right), \tag{19}$$

where $F\left(a,b,c;x\right)$ is the hypergeometric Gaussian function. The use of relations for the adjacent hypergeometric functions yields

$$Q\left(r\right)=\frac{3\left(\alpha/8\right)^2}{2\pi}QF\left(1,5/2,1;-\left(\alpha r/8\right)^2\right). \tag{20}$$

Further, for the function $F\left(1,5/2,1;x\right)$, we use the following relation:

$$F\left(1,5/2,1;x\right)=\frac{4}{3}x^2\frac{d^2}{dx^2}\left(x^{3/2}F\left(1/2,1,1;x\right)\right), \tag{21}$$

and, allowing for the fact that

$$F\left(1/2,1,1;x\right)=\left(1-x\right)^{-1/2}, \tag{22}$$

we finally get

$$Q(r)=\frac{3}{2\pi}\frac{Q\left(\alpha/8\right)^2}{\left(1+\left(\alpha r/8\right)^2\right)^{5/2}}. \tag{23}$$

Substituting Eq. (23) into the integral in the right side of equality (3), we can see its validity. The charge-distribution density obtained is finite and is increased as r^{-5} with the distance from the center of the particle, which is physically acceptable. It is also obvious that the particle-field energy with the charge distribution (23) is largely concentrated in the region $r \le 8/\alpha$.

6. Summing up and discussing the results obtained, we note the following:

– A charge form factor of a particle in a three-dimensional electrodynamics is proposed that can be used for deriving relations for the potential and field strength of the particle having asymptotic behavior at $r\to\infty$. This behavior coincides with the behavior of the corresponding values in the case of the delta form factor.

– The form factor proposed allows the electrical charge distribution within the particle to be found in a closed form (equation (23)).

– On the basis of the form factor , it was established that the tension-tensor components of a field generated by a charged particle turn to zero in a three-dimensional electrodynamics with vacuum polarization by massless fermions taken into account; this circumstance allows us to put forward a hypothesis on the dynamic origin of particle mass in the case under study.

– The field particle mass is determined in a three-dimensional electrodynamics. The value is finite, with $m \sim \alpha$ (equation (17)), which, as was pointed out above, is natural for the model under consideration.

Before proceeding to discussion of the results obtained, let us examine the extent to which they are reliable from the point of view of their dependence, first, on a particular form factor, second, on improvements in the approximation, and, third, on the transition to pure quantum considerations.

Dealing with the dependence of the results on a particular form factor, we note that of critical importance here is the behavior of field strength at $r\to\infty$, since the singularity at a zero point can be eliminated by introducing an electric particle radius [9, 10]. As for the sharp truncation at $r\to\infty$, this can hardly be assigned the same clear meaning to. On the other hand, it follows from Eq. (15) that in the case where the integral is truncated in the upper limit equal $\alpha/8$, we get Eq. (17). Therefore, the only requirement to the form factor, that is, a decrease in the potential and field strength with $r\to\infty$, seems to be justified in this case; another form of the form factor results only in a change in the numerical coefficient for α in Eq. (17) when the requirement is fulfilled.

Speaking of improvements in the approximation, we point, first and foremost, to the allowance for mass made in the polarization operator, the importance of which was emphasized in [4, 13]. Indeed, this leads to a change in the asymptotic behavior of $A_0\left(r\right)$ at $r\to\infty$ [4, 13], and, hence, the specific form of form factor (9) must change as follows:

$$\varphi\left(k\right)=8Q\frac{k^2+i\Pi_R}{\alpha k}e^{-8k/\alpha}. \tag{24}$$

Finally, let us compare our results for a dynamic mass with those obtained from quantum calculations. In the quantum case, we have [8]

$$m_q \approx (\alpha/8) e^{-3\pi^2 N/32} . \tag{25}$$

Using $N \approx 3$ [2 – 4] for N, we get

$$m_q \approx 7.9 \cdot 10^{-3} \alpha . \tag{26}$$

Classical formula (17) results in

$$m \approx 6.6 \cdot 10^{-3} \alpha . \tag{27}$$

Thus, both in the first and second case, the inequality $m << \alpha$ is observed with reasonable accuracy and the masses m and m_q are of similar order.

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