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MOTION OF A CHARGED PARTICLE IN THE FIELD OF A GROUNDED CONDUCTIVE SPHERE

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Motion of a charged particle in the field of a grounded conductive sphere is investigated. It is assumed that the field created by the sphere is quasi-static that implies limitation on particle velocities by nonrelativistic values and the possibility of neglecting magnetic interaction and heat losses. A classification is provided and possible particle trajectories are constructed in the examined case.

Keywords: charged particle, grounded sphere, trajectory equation, effective potential energy.

The charged particle interaction with central fields is described in ample detail in the scientific literature [1–4]. Along with this, continuation of investigations in this direction has recently yielded radically new results [5–9]. However, these results refer to the static case when the particle in an external field is at rest. In this work, the situation when the particle moves in the field of a grounded conducting sphere is considered. We note that this problem is simpler on the one hand and can be solved by standard methods [10, 11]. On the other hand, it is sufficiently urgent, since its solution can be used to construct different models in other fields of physics. Moreover, it can have practical significance, for example, for the development and improvement of electric methods of mineral enrichment [12].

1. We now examine the case of nonrelativistic particle velocities. This leads to the quasi-static nature of the field in which this sphere moves and allows us to neglect the presence of magnetic field and heat losses in the vicinity of the sphere surface. This circumstance allows the electrical image method to be used and the problem to be reduced to investigation of the interaction of two point charges – real and imaginary ones. Moreover, the imaginary charge and the charge spacing depend on the real charge coordinates $\left(q' = -q\frac{R}{r} \right.$ and $a = \frac{R^2}{r}$, where q and q' are the real and imaginary charges, respectively; R is the sphere radius; r is the distance from the sphere center to the real charge, and a is the distance from the sphere center to the imaginary charge $\left. [2] \right)$.

With allowance for the foregoing, we represent the expression for the particle energy in the following form:

$$E = -\frac{1}{4\pi\varepsilon_0}\frac{q^2 R}{2\left(r^2 - R^2\right)} + \frac{m\mathrm{v}^2}{2}. \tag{1}$$

Equation (1) demonstrates that the field is central. Therefore, as usual for analogous problems [7], we take advantage of the law of conservation of particle angular momentum L relative to the sphere center and introduce the effective potential charge energy U_{eff}:

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$$U_{\text{eff}} = -\frac{1}{4\pi\varepsilon_0}\frac{q^2 R}{2\left(r^2 - R^2\right)} + \frac{L^2}{2mr^2}. \tag{2}$$

Below we introduce the dimensionless variables λ_0, u, and ξ determined by the relationships

$$\lambda_0 = \frac{K_0}{U_0} = \frac{2\pi\varepsilon_0 L^2}{q^2 mR},\ u = \frac{U_{\text{eff}}}{U_0},\ \xi = \frac{r}{R}, \tag{3}$$

where $U_0 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{R}$ and $K_0 = \frac{L^2}{2mR^2}$. In these variables, the dimensionless effective potential energy $u(\xi)$ assumes the form

$$u(\xi) = -\frac{1}{2\left(\xi^2 - 1\right)} + \frac{\lambda_0}{\xi^2}. \tag{4}$$

Investigation of the function $u(\xi)$ gives the following results:

a) $\lim\limits_{\xi\to 1} u(\xi) = -\infty$, $\lim\limits_{\xi\to\infty} u(\xi) = 0$;

b) The plot $u(\xi)$ intersects the *Ox* axis once at the point $\xi_1 = \sqrt{\frac{2\lambda_0}{2\lambda_0 - 1}}$ if $\lambda_0 > \frac{1}{2}$ and does not intersect it if $\lambda_0 \le \frac{1}{2}$;

c) The function has one maximum at the point $\xi_2 = \sqrt{\frac{\sqrt{2\lambda_0}}{\sqrt{2\lambda_0} - 1}}$ if $\lambda_0 > \frac{1}{2}$ and increases monotonically if $\lambda_0 \le \frac{1}{2}$.

The results obtained demonstrate that there is no stable equilibrium position for the particle; in fact, the effective energy has no minimum.

2. The results obtained above can be used for the classification of possible particle trajectories. Indeed, as it follows from Eqs. (1) and (2), the solution of equation

$$E = U_{\text{eff}} \tag{5}$$

for r determines motion boundaries. Introducing the dimensionless energy $w = E/U_0$, we write Eq. (5) in the form

$$w = u(\xi), \tag{6}$$

where $u(\xi)$ is determined by Eq. (4). Exactly Eq. (6) is the subject of our further investigation.

We now consider possible solutions of Eq. (6) for ξ taking into account the condition $\xi \ge 1$.

Case 1. $\lambda_0 \le 1$:

a) For $w < 0$, the equation has one root:

$$\xi_1 = \frac{1}{2}\sqrt{\frac{X - \sqrt{X^2 - 16w\lambda_0}}{w}} . \tag{7}$$

Here $X = 2w + 2\lambda_0 - 1$;

b) For $w \geq 0$, it has no roots.

Case 2. $\lambda_0 > 1$:

a) For $w \leq 0$, it has one root coinciding with Eq. (7);

b) For $0 < w < \frac{\left(\sqrt{2\lambda_0} - 1\right)^2}{2}$, it has two roots:

$$\xi_2 = \frac{1}{2}\sqrt{\frac{X - \sqrt{X^2 - 16w\lambda_0}}{w}} , \tag{8}$$

$$\xi_3 = \frac{1}{2}\sqrt{\frac{X + \sqrt{X^2 - 16w\lambda_0}}{w}} ; \tag{9}$$

c) For $w = \frac{\left(\sqrt{2\lambda_0} - 1\right)^2}{2}$, it has one root:

$$\xi_4 = \frac{1}{2}\sqrt{X/w} ; \tag{10}$$

d) For $w > \frac{\left(\sqrt{2\lambda_0} - 1\right)^2}{2}$, it has no roots.

Therefore, in our further investigation we consider seven different cases of particle motion. The results obtained will be used to construct trajectories corresponding to these cases.

3. We obtain the equation of the charge trajectory in polar coordinates using the standard methods [13]. Choosing the origin of coordinates in the sphere center, counting the polar angle from the straight line joining the sphere center and the point charge at the initial time moment, and using the introduced dimensionless parameters, we write down

$$\varphi(\xi) = \sqrt{\lambda_0} \int \frac{d\xi}{\xi^2 \sqrt{w + \frac{1}{2\left(\xi^2 - 1\right)} - \frac{\lambda_0}{\xi^2}}} . \tag{11}$$

The integral entering into Eq. (11) is not expressed through known special elementary functions. Thus, in the present work the right side of Eq. (11) is integrated numerically for different values of the parameters entering into the integral; in this case, the integration limits are changed according to the motion boundaries calculated above (see Eqs. (7)–(10)). For example, for $\lambda_0 \leq \frac{1}{2}$ and $w < 0$, we have

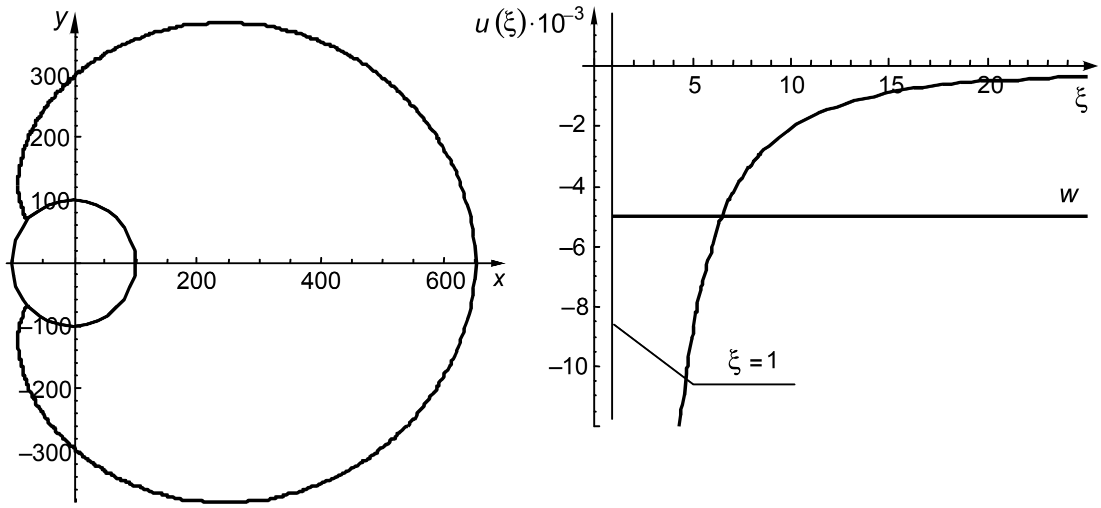

Fig. 1. Dependences $y(x)$ and $u(\xi)$ for $\lambda_0 = 0.3$ and $w = -0.005$.

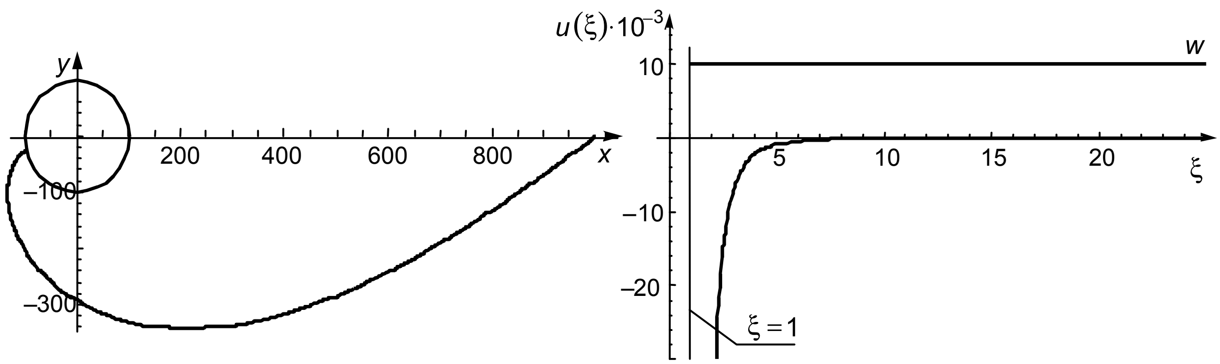

Fig. 2. Dependences $y(x)$ and $u(\xi)$ for $\lambda_0 = 0.3$ and $w = 0.01$.

$$\varphi(\xi) = \sqrt{\lambda_0} \int_1^{\xi_0} \frac{d\xi}{\xi^2 \sqrt{w + \frac{1}{2(\xi^2 - 1)} - \frac{\lambda_0}{\xi^2}}}, \; 1 \le \xi_0 \le \xi_1,$$

where ξ_1 is determined by Eq. (8).

Thus, for each of the seven cases, we obtain the table of $\varphi(\xi)$ correspondence to fixed λ_0 and w values and preset initial conditions. This allows us to construct the trajectories in the Cartesian coordinates. Some examples are shown in Figs. 1–6. Numerical calculations and graphical constructions were done using the software package Mathematica 7.0.

1) $\lambda_0 \le \frac{1}{2}$ and $w < 0$.

2) $\lambda_0 \le \frac{1}{2}$ and $w > 0$.

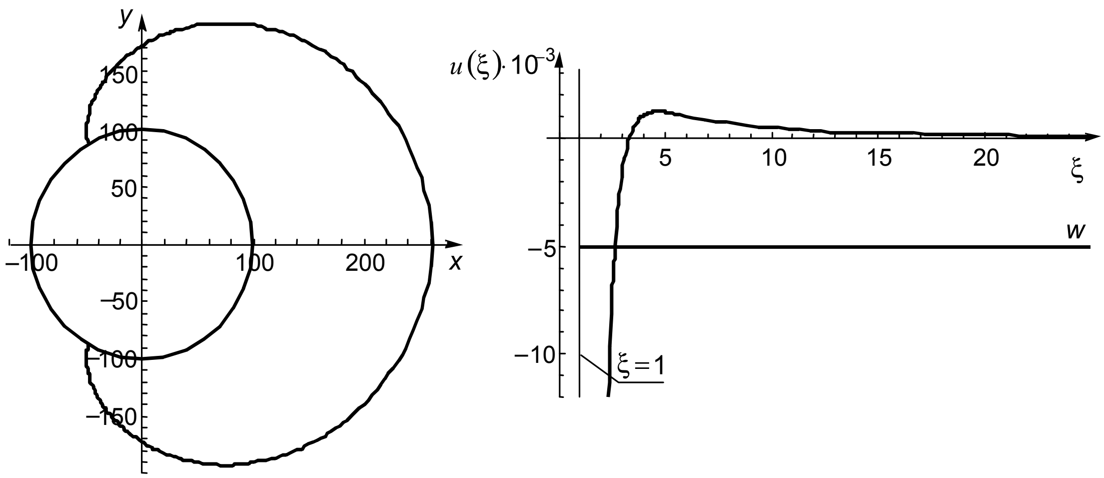

Fig. 3. Dependences $y(x)$ and $u(\xi)$ for $\lambda_0 = 0.55$ and $w = -0.005$.

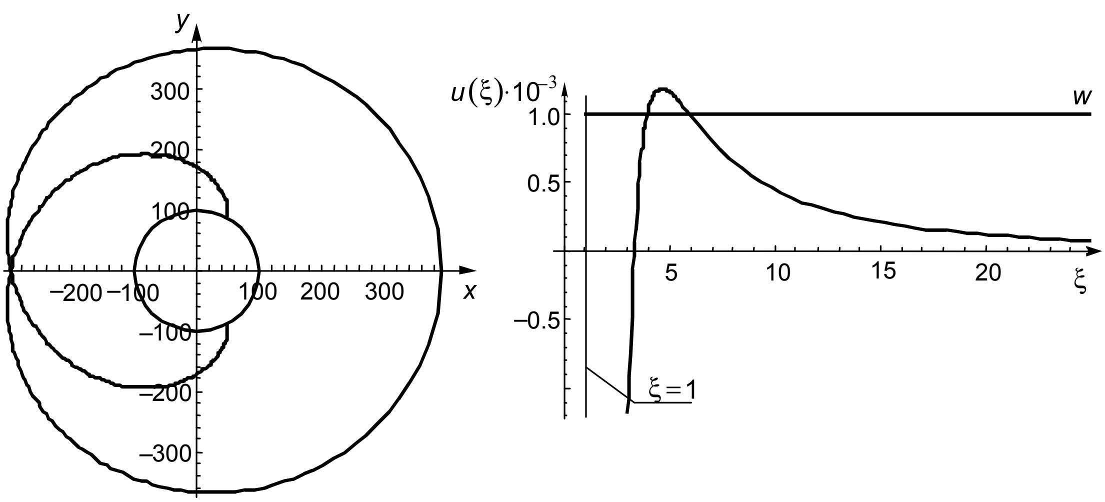

Fig. 4. Dependences $y(x)$ and $u(\xi)$ for $\lambda_0 = 0.55$, $w = 0.001$, and $\xi \le \xi_2$.

3) $\lambda_0 > \frac{1}{2}$ and $w < 0$.

4) $\lambda_0 > \frac{1}{2}$, $0 < w < \frac{\left(\sqrt{2\lambda_0} - 1\right)^2}{2}$, and $\xi \le \xi_2$.

5) $\lambda_0 > \frac{1}{2}$, $0 < w < \frac{\left(\sqrt{2\lambda_0} - 1\right)^2}{2}$, and $\xi \ge \xi_3$.

6) $\lambda_0 > \frac{1}{2}$ and $w > \frac{\left(\sqrt{2\lambda_0} - 1\right)^2}{2}$.

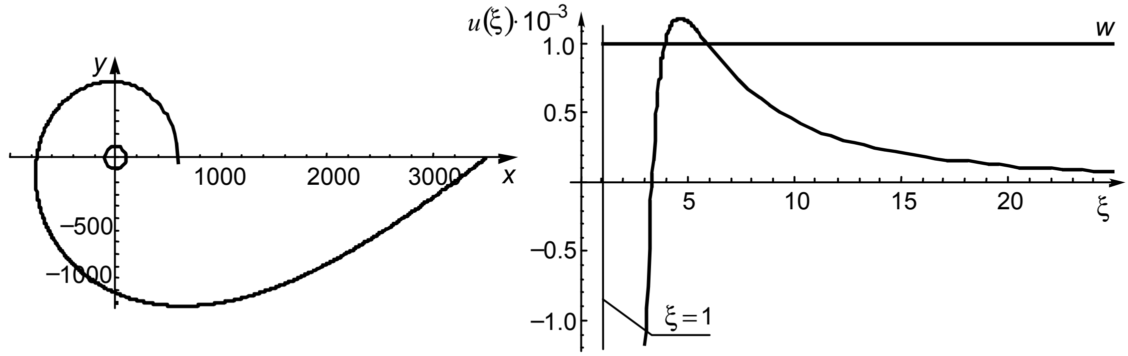

Fig. 5. Dependences $y(x)$ and $u(\xi)$ for $\lambda_0 = 0.55$, $w = 0.001$, and $\xi \geq \xi_3$.

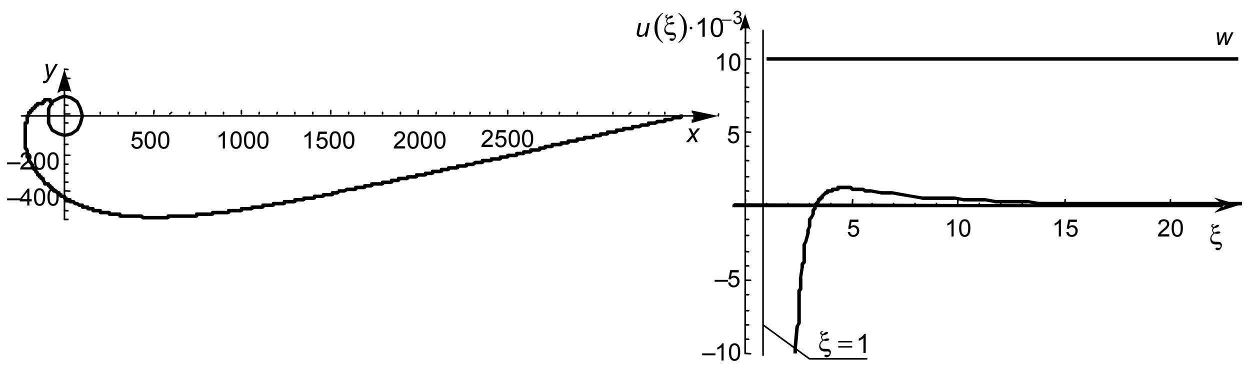

Fig. 6. Dependences $y(x)$ and $u(\xi)$ for $\lambda_0 = 0.55$ and $w = 0.01$.

4. We now summarize main results of the work. The motion of the charged particle in the grounded conductive sphere field has been investigated under assumptions indicated in Section 1. The motion trajectories were classified in the examined case. It was demonstrated that:

a) The particle trajectory depends only on two dimensionless parameters λ_0 and w described above and the initial conditions;

b) In terms of the above-indicated parameters, possible particle trajectories were classified and it was demonstrated that seven different cases of motion existed; the corresponding graphical dependence $u(\xi)$ and $y(x)$ were drawn for each case (see Eq. (4));

c) The particle trajectory was always a plane curve;

d) Among all possible particle trajectories, closed curves were absent excluding the case of the circle with radius $R_0 = \frac{R}{2}\sqrt{X/w}$ (see Eq. (10)) which corresponded to the unstable equilibrium state.

We note also that some trajectories are visually similar (for example, Figs. 1, 3, and 4 or Figs. 2, 5, and 6). This gives us grounds to believe that the examined classification must be investigated more carefully; in particular a wider spectrum of initial conditions should be considered for a more detailed classification of trajectories.

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