Russian Physics Journal, Vol. 52, No. 1, 2009

PHASE TRANSITION DRIVEN BY THE COUPLING CONSTANT IN QED_3 IN THE PRESENCE OF THE ULTRAVIOLET CUTOFF

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Keywords: phase transition, coupling constant, dynamic mass, cutoff factor.

1. Dynamic symmetry breaking is a nonperturbative process; therefore, to solve the Schwinger–Dyson equations to study this process, methods other than those based on perturbation theory are required. Thus, in [1–4] it was revealed that already in the simplest nonperturbative ladder approach for the electron propagator in QED_4, an originally massless electron acquires a dynamic mass when the coupling constant exceeds its critical value. More recently, it has been demonstrated that the main features of the phenomenon established in the ladder approximation are also retained beyond its limits [5].

The similar situation is also typical of QED_3, except the absence of the parameter whose threshold value would indicate the phase transition accompanied by dynamic chiral symmetry breaking. On the other hand, an interesting and, apparently, far-reaching analogy between QED_4 at a nonzero temperature (with the temperature being the parameter of ultraviolet cutoff) and QED_3 (with the corresponding parameter at zero temperature) has recently been established [6]. This circumstance stimulates a more detailed study of the above-indicated model to establish the presence of the above-mentioned transition and the corresponding threshold parameter.

2. We proceed from the Schwinger–Dyson equation for the fermion mass function in the ladder approximation with the Landau gauge for the photon propagator. We have [1, 2]

$$M(p)=\frac{2e_0^2}{(2\pi)^3}\int\frac{d^3k}{(p-k)^2}\frac{M(k)}{k^2+M^2(k)}, \tag{1}$$

where e_0^2 is the dimensional coupling constant in QED_3, p and k are three-dimensional Euclidean vectors, and M is the mass function of interest to us. Integrating Eq. (1) over angles, we obtain

$$M(p)=\frac{2e_0^2}{(2\pi)^2}\left(\frac{1}{p}\int_0^\infty kdk\frac{M(k)}{k^2+M^2(k)}\ln\left|\frac{p+k}{p-k}\right|\right). \tag{2}$$

Chiral symmetry breaking in the examined model is manifested, in particular, through the existence of a nonzero solution of Eq. (1) (or Eq. (2) equivalent to it).

We now make the following assumptions. The first assumption is the applicability of a sharp cutoff of the upper integration limit in Eq. (2) by introducing the limiting momentum $k_{\max}$, and the second assumption is the presence of the proportionality

$$e_0^2=\alpha k_{\max}, \tag{3}$$

where α is a dimensionless parameter. In addition, we consider that in the vicinity of the critical value of the parameter α in the denominator of Eq. (2), the term comprising M^2 can be neglected in comparison with k^2. Finally, introducing

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an infrared cutoff in addition to the ultraviolet one and choosing the Euclidean mass m as a parameter of the infrared cutoff, we write down

$$M(p) = \frac{2\alpha k_{\max}}{(2\pi)^2}\left(\frac{1}{p}\int_m^{k_{\max}} dk \frac{M(k)}{k}\ln\left|\frac{p+k}{p-k}\right|\right). \tag{4}$$

We further take advantage of the standard methods. We express the logarithm in formula (4) as

$$\ln\left|\frac{p+k}{p-k}\right| = 2\sum_{n=0}^{\infty}\frac{1}{2n+1}\left(\theta(p-k)(p/k)^{2n+1} + \theta(k-p)(k/p)^{2n+1}\right) \tag{5}$$

and consider the first terms of the sum in the above expansion. Then we obtain the Volterra integral equation with degenerate kernel

$$M(p) = \frac{\alpha k_{\max}}{\pi^2}\left(\frac{1}{p^2}\int_m^p M(k)\,dk + \int_p^{k_{\max}} M(k)\frac{dk}{k^2}\right), \tag{6}$$

equivalent to the differential equation

$$\frac{d^2M}{dp^2} + \frac{3}{p}\frac{dM}{dp} + \frac{2\alpha k_{\max}}{p^3}M(p) = 0 \tag{7}$$

with boundary conditions

$$M(m) = m, \quad \left.\frac{dM}{dp}\right|_{p=m} = 0, \tag{8}$$

where m is determined by the relation

$$m = \frac{\alpha k_{\max}}{\pi^2}\int_m^{k_{\max}} M(k)\frac{dk}{k^2}. \tag{9}$$

3. A solution of Eq. (7) satisfying conditions (8) has the form

$$\begin{aligned} M(p) = \frac{4\pi m^2}{p}\left(\frac{\alpha k_{\max}}{8\pi^2 m}\right)^{1/2} & \left(J_1\left(\left(8\alpha k_{\max}/\pi^2 m\right)^{1/2}\right)\cdot\left(N_2\left(\left(8\alpha k_{\max}/\pi^2 p\right)^{1/2}\right)\right.\right. \\ & \left.\left. - N_1\left(\left(8\alpha k_{\max}/\pi^2 m\right)^{1/2}\right)\cdot J_2\left(\left(8\alpha k_{\max}/\pi^2 p\right)^{1/2}\right)\right)\right). \end{aligned} \tag{10}$$

Here J and N are the Bessel and Neumann functions of the corresponding order. Substituting this solution into Eq. (9), we obtain

$$J_1\left(\left(8\alpha/\pi^2\right)^{1/2}\right)\cdot N_3\left(\left(8\alpha k_{\max}/m\right)^{1/2}\right) - N_1\left(\left(8\alpha/\pi^2\right)^{1/2}\right)\cdot J_3\left(\left(8\alpha k_{\max}/m\right)^{1/2}\right) = 0. \tag{11}$$

The above equality should be considered as an equation for α at which it has solutions satisfying the condition $m/k_{\max} \ll 1$. Numerical solution of this equation allows a minimal value of the given parameter – α_{cr} – to be calculated at which the above-indicated solutions do exist. For this parameter value, we have

$$\alpha_{cr} = 4\pi^2 \approx 39.478 . \tag{12}$$

Numerical calculations of Eqs. (2) and (4) with the help of the program package *Mathematica 5.1* yield

$$\alpha_{cr} = \begin{cases} 37.130 \text{ for Eq. (2)}, \\ 35.974 \text{ for Eq. (4)}. \end{cases} \tag{13}$$

Thus, the cutoff factor allows the dynamic mass to be taken into account in the ladder approximation in QED_3 as a phase transition driven by the coupling constant α with the transition point specified by Eqs. (12) and (13). In this case, the transition point α_{cr} in the ladder approximation depends slightly on the accuracy of the approximation used for solving main equation (2). On the other hand, since $\alpha_{cr} > 1$, we note that a comparatively small region of momenta in the vicinity of $p \sim m$ contributes to the corresponding integrals in Eqs. (2) and (4). This circumstance considerably differs from the case of QED_4 [2], where this contribution is determined by the region $p \sim k_{cr}$.

In our opinion, of interest is further consideration of this problem, in particular, for the given model beyond the limits of the ladder approximation, its dependence on the gauge, and so on.

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