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RELATIVISTIC FERMION IN A SPHERICALLY SYMMETRIC POTENTIAL WELL OF FINITE DEPTH IN A TWO-DIMENSIONAL SPACE

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The problem of existence of bounded relativistic fermion states in a spherically symmetric well of finite depth in a two-dimensional space is investigated. The well depth critical for the appearance of standard states with energies $E = m$, 0, and $-m$ is determined; moreover, cases with zero and nonzero fermion momenta are considered. Approximate analytical expressions for the critical depths of narrow and wide wells are derived which are in good agreement with the results of numerical calculations. Approximate energies of levels located on the boundaries of the upper and lower continuums and determined analytically are in good agreement with the results of numerical calculations.

Keywords: spherically symmetric potential well, bound states, two-dimensional space, critical well depth.

The problem of existence of bound relativistic fermion states in a spherically symmetric well of finite depth in a two-dimensional space is considered. Interest in the study of this problem is caused by the following circumstances. First of all, considerable recent attention is given to the study of various two-dimensional systems like single-atomic graphite layer – graphene [1, 2]; problems of investigating the quantum Hall effect can also be mentioned here [3–5]. On the other hand, breaking of chiral symmetry accompanied by the appearance of the dynamic particle mass takes place in quantum field theory; in this case, a close analogy is observed between the processes of particle incidence in the center of a strong field in the one-particle problem of relativistic quantum mechanics and the dynamic particle mass appearing in the quantum field approach [6]. In this respect, quantum electrodynamics of three-dimensional space (QED_{2+1}) represents an example of a quantum field model in which, in a certain approach, breaking of the chiral symmetry studied in ample detail in [7–9] is observed. This is not the case for the corresponding two-dimensional relativistic quantum mechanical problem that was studied only in a few works (for example, see [10, 11]); moreover, a number of problems studied in details in the spatial case [12–14] remained uninvestigated for its two-dimensional analog.

In the nonrelativistic case, certain peculiarities were observed in the formation of bound states in the potential well of finite depth when the number of dimensions changed [15]. It is of interest to elucidate the degree of peculiarity that is retained in the relativistic case. There are also some other reasons that justify the consideration of the above-indicated model.

The purpose of the present work is to determine for the two-dimensional space the depth of the spherically symmetric potential well critical for the appearance of bound states corresponding to standard energies of the relativistic fermion indicated below. The choice of the examined object here is caused by the comparatively simple model and the possibility of comparison of the results of approximate analytical calculations with the data of subsequent numerical calculations. In future, we plan to investigate the structure of the energy levels in the examined case, to consider the possibility of appearance of stationary states in the well whose depth exceeds the critical one, and to investigate more realistic two-dimensional models.

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1. We proceed from the two-dimensional stationary Dirac equation

$$\hat{H}\Psi = E\Psi \,. \tag{1}$$

Here $\hat{H} = \boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta m + V(r_\perp)$ is the Dirac Hamiltonian, where $V(r_\perp) = -V_0 \cdot \theta(r_{\perp 0} - r_\perp)$ and θ is the Heaviside function ($r_\perp$ is the modulus of the two-dimensional vector $\boldsymbol{r}_\perp$; the subscript of the vector is omitted below). For matrices $\boldsymbol{\alpha}$ and β we choose two-dimensional representations $\boldsymbol{\alpha} = (\sigma_1, \sigma_2)$ and $\beta = \sigma_3$ (here σ_1, σ_2, and σ_3 are the Pauli matrices). The operator $\hat{J}_z = \hat{L}_z + \hat{S}_z$ $\left(\hat{L}_z = -i\dfrac{\partial}{\partial\varphi} \text{ and } \hat{S}_z = \dfrac{1}{2}\sigma_3\right)$ commutes with the Hamiltonian; hence, Ψ is the common eigenfunction for the indicated operators. This allows us to represent a solution of Eq. (1) in the form

$$\Psi(\boldsymbol{r}) = e^{il\varphi}\begin{pmatrix}\Psi_1(r)\\ i\Psi_2(r)e^{i\varphi}\end{pmatrix}, \tag{2}$$

where $l = j_z \mp 1/2 = 0; \pm 1; \pm 2 \ldots$ are eigenvalues of the operator $\hat{L}_z$ and j_z are eigenvalues of the total momentum.

Taking advantage of Eqs. (1) and (2), for the function Ψ_1 we obtain the equation

$$\frac{d^2\Psi_1}{dr^2} + \frac{1}{r}\frac{d\Psi_1}{dr} - \left(\frac{l^2}{r^2} - \left((E+V_0)^2 - m^2\right)\right)\Psi_1 = 0\,, \tag{3}$$

whose solution, finite at the origin of coordinates and at infinity, has the form

$$\begin{aligned}\Psi_1(r) &= AJ_{|l|}(kr), \quad \text{for} \quad r < r_0,\\ \Psi_1(r) &= BK_{|l|}(\kappa r), \quad \text{for} \quad r > r_0,\end{aligned} \tag{4}$$

where $k = ((E+V_0)^2 - m^2)^{1/2}$ and $\kappa = (m^2 - E^2)^{1/2}$. For the function Ψ_2 we have

$$\begin{aligned}\Psi_2(r) &= A\frac{k}{E+V_0+m}J_{|l|+1}(kr), \quad \text{for} \quad r < r_0,\\ \Psi_2(r) &= B\frac{\kappa}{E+m}K_{|l|+1}(\kappa r), \qquad \text{for} \quad r > r_0.\end{aligned} \tag{5}$$

In Eqs. (4) and (5), J and K are the Bessel and McDonald functions of the indicated order, respectively.

For the functions Ψ_1 and Ψ_2 on the well boundary, the relationships

$$(\Psi_1/\Psi_2)_{r\to r_0-0} = (\Psi_1/\Psi_2)_{r\to r_0+0} \tag{6}$$

are fulfilled, from which the eigenvalues of the particle energy in the examined well [12] are determined. Substituting Eqs. (4) and (5) into Eq. (6) and introducing dimensionless variables $\varepsilon = E/m$, $v_0 = V_0/m$, $\chi' = (1-\varepsilon^2)^{1/2}$, $k' = (v_0^2 + 2\varepsilon v_0 - \chi'^2)^{1/2}$, and $\xi = mr_0$, we obtain

$$\left(\frac{\varepsilon+v_0+1}{\varepsilon+v_0-1}\right)^{1/2}\frac{J_{|l|}(k'\xi)}{J_{|l|+1}(k'\xi)} = \left(\frac{1+\varepsilon}{1-\varepsilon}\right)^{1/2}\frac{K_{|l|}(\chi'\xi)}{K_{|l|+1}(\chi'\xi)}\,. \tag{7}$$

This expression is the key one for the present work. For the three-dimensional well, analogous expression was derived and studied in [12]. A formal distinction between cases considered here and in [12] consists primarily in the fact that the fermion Ψ functions for the three-dimensional well are expressed through elementary functions, which cannot be done in our case.

2. Formula (7) can be considered not only as an equation for particle energy in the well, but also as an expression for the minimum well depth V_{0c} at which a level with fixed energy appears. We now consider energies typical for our problem: $E = m$ (lower boundary of the upper continuum), $E = 0$, and $E = -m$ (upper boundary of the lower continuum).

a) Level $\varepsilon = 1$. We consider the appearance of the *s*-level $(|l| = 0)$ assuming that it penetrates insignificantly into the well depth, that is, $\varepsilon = 1 - x$ and $x << 1$. Expanding the McDonald functions $K_n(z)$ for $z \to 0$ and considering the first terms of expansions

$$K_0(z) = -(\ln(z/2) + C), \quad K_n(z) = (1/2)\Gamma(n)(z/2)^{-n} \text{ for } n \geq 1, \tag{8}$$

we obtain

$$x = \left(2/(\xi^2\gamma)\right)e^{-\frac{F_0(v_0)}{\xi}}. \tag{9}$$

In Eq. (9), we have used the following designations: $C = 0.577\ldots$ is the Euler constant, $\gamma = e^C = 1.781...$, and $F_0(v_0) = \left(\frac{v_0 + 2}{v_0}\right)^{1/2} \frac{J_0((v_0^2 + 2v_0)^{1/2}\xi)}{J_1((v_0^2 + 2v_0)^{1/2}\xi)}$.

Condition $x << 1$ must be fulfilled for the shallow well ($v_0\xi << 1$). Considering the first terms in the expansion of the Bessel functions $J_n(z)$ for $z \to 0$, we obtain

$$F_0(v_{0c}) = 2/(v_{0c}\xi). \tag{10}$$

The result obtained does not practically differ from that for the nonrelativistic case [15]. In particular, from this result it follows that the critical depth here is $v_{0c} = 0$, that is, the bound *s*-state of the relativistic fermion appears at any arbitrary depth in the examined well. In this respect, the two-dimensional case differs radically from the three-dimensional one in which, as is well known, $v_{0c} \neq 0$ both for nonrelativistic [15] and relativistic particles [12].

Let us consider the appearance of the level $\varepsilon = 1$ for $|l| \neq 0$ and find v_{0c} for this case. Here for $\varepsilon = 1$ we have

$$F_l(v_{0c}) = \xi/|l|. \tag{11}$$

We further consider separately cases with $\xi << 1$ (narrow well) and $\xi >> 1$ (wide well).

Case $\xi << 1$. We seek v_{0c} in the form

$$v_{0c}(\xi) = a_0/\xi + a_1(\xi), \tag{12}$$

where $a_1(\xi)$ is a function regular at $\xi = 0$. Considering the principal term of the expansion of the function $a_1(\xi)$ in powers of ξ, we obtain $a_0 = s_l$ and $a_1(\xi) = (1/|l|)\left(J_{|l|+1}(s_l)/J'_{|l|}(s_l)\right) - 1$, where s_l is the first nonzero root of the function $J_{|l|}$. Thus, considering two principal terms of v_{0c} expansion in powers of ξ and taking advantage of the recurrent relations for the Bessel functions, we obtain

$$v_{0c} = (s_l/\xi) - (1/|l|)(1 + |l|). \tag{13}$$

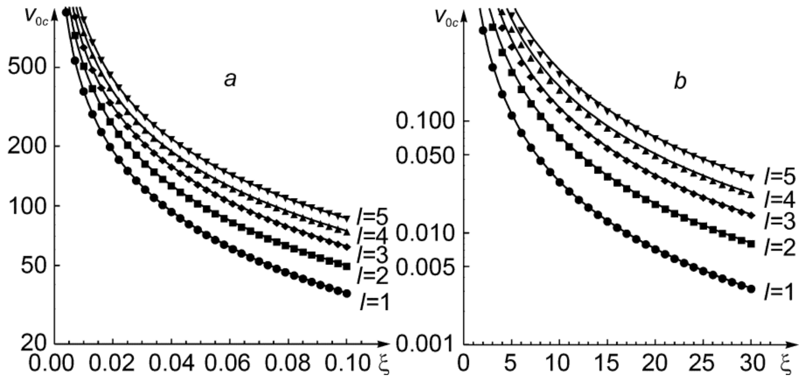

Fig. 1. Dependence of v_{0c} on ξ at ε = 1 and indicated l values for narrow (a) and wide wells (b). Solid curves are for the approximate formulas, and closed circles are for numerical solutions of the exact equation.

Let us consider the case when $\xi >> 1$. We seek the function $v_{0c}(\xi) \to 0$ for $\xi \to \infty$ in the form

$$v_{0c}(\xi) = A_l / \xi^2 . \tag{14}$$

Then, considering in Eq. (11) the principal terms for $\xi \to \infty$, we obtain

$$2l J_{|l|}(z) / J_{|l|+1}(z) = z , \tag{15}$$

where $z = (2A_l)^{1/2}$. It is easy to demonstrate that the roots of Eq. (15) coincide with the first nonzero roots of the function $J_{|l|-1}(z)$, that is, A_1 = 2.88, A_2 = 7.33, A_3 = 13.22, and so on.

Figure 1 demonstrates good qualitative and satisfactory quantitative agreements of the results calculated from formulas (13) and (14) with the numerical solution of Eq. (11). Thus, we can consider that Eqs. (13) and (14) determine correctly the critical depths of the narrow and wide wells for $|l| \neq 0$.

b) Level ε = 0. Designating $k' = (v_{0c}^2 - 1)^{1/2}$, we reduce Eq. (7) to the form

$$\left(\frac{v_{0c}+1}{v_{0c}-1}\right)^{1/2} \frac{J_{|l|}(k'\xi)}{J_{|l|+1}(k'\xi)} = \frac{K_{|l|}(\xi)}{K_{|l|+1}(\xi)} . \tag{16}$$

Because analytical solution of Eq. (16) is impossible in general case, we consider narrow and wide wells separately again. We first consider the appearance of the s-level $(|l| = 0)$ in the narrow well. Taking advantage of expansion (8), for Eq. (7) we obtain

$$\left(\frac{v_{0c}+1}{v_{0c}-1}\right)^{1/2} \frac{J_0(k'\xi)}{J_1(k'\xi)} = -\xi \cdot \ln\left(\frac{\gamma\xi}{2}\right), \tag{17}$$

which prompts us that roots of this equation for $\xi << 1$ are close to $k'\xi = s_{0n}$, where s_{0n} are roots of the function $J_0(z)$. We are interested in a minimal root of this equation. Then it is expedient to represent the argument of the Bessel function in Eq. (17) in the form

$$(v_{0c}^2 - 1)^{1/2} \xi = s_0 + f(\xi) , \tag{18}$$

where the function $f(\xi)$ for $\xi << 1$ satisfies the condition

$$f(\xi) << s_0 . \tag{19}$$

Solving Eq. (18) for the function $v_{0c}(\xi)$, substituting the solution obtained into equality (17), and considering the principal terms of expansion of the function $v_{0c}(\xi)$ in $f(\xi)$ and ξ, we find

$$f(\xi) = \xi \ln(\gamma\xi/2) . \tag{20}$$

From Eq. (20) it can be seen that the assumption that inequality (19) is satisfied is valid for $\xi << 1$. As a result, in the case under consideration we have for the critical depth

$$v_{0c}(\xi) = s_0/\xi + \ln(\gamma\xi/2) . \tag{21}$$

Let us solve the problem for $|l| \neq 0$. For the narrow well $(\xi << 1)$, we take advantage of expansion (8); therefore, Eq. (16) for the level $\varepsilon = 0$ assumes the form

$$\left(\frac{v_{0c}+1}{v_{0c}-1}\right)^{1/2} \frac{J_{|l|}(k'\xi)}{J_{|l|+1}(k'\xi)} = \frac{1}{2}\frac{\xi}{|l|} . \tag{22}$$

By analogy with the case of the *s*-level, expression for $\left(v_{0c}^2-1\right)^{1/2}\xi$ here can be written in the form

$$(v_{0c}^2-1)^{1/2}\xi = s_l + f_1(\xi) \tag{23}$$

(see Eq. (18)), where s_l is the minimal nonzero root of the function $J_{|l|}(z)$ and $f_1(z)$ is the function satisfying the analog of inequality (19): $f_1(\xi) << s_l$. By analogy with the case described above, considering the term principal for ξ/s_l, we obtain $f_1(\xi) = -(2|l|)^{-1}\xi$, from which we obtain the critical well depth with the required accuracy:

$$v_{0c}(\xi) = s_l/\xi - 1/(2|l|) . \tag{24}$$

From Fig. 2 it can be seen that Eqs. (21) and (24) (solid curves in the figure) coincide with high accuracy with numerical solutions of Eqs. (17) and (22) (closed circles in the figure), respectively.

The wide well $(\xi >> 1)$ can be considered both for $l = 0$ and $|l| \neq 0$. To this end, we take advantage of asymptotic representations of the Bessel and McDonald functions for $z >> 1$ and any arbitrary *l* values:

$$J_{|l|}(z) = (2/\pi z)\cos\left(z - (\pi|l|/2) - (\pi/4)\right),\ K_{|l|}(z) = 1 .$$

In this case, Eq. (7) is reduced to the form

$$\left(\frac{v_{0c}+1}{v_{0c}-1}\right)^{1/2} \frac{1+\eta\cdot\tan(k'\xi)}{\tan(k'\xi)-\eta} = 1 , \tag{25}$$

where $\eta = +1$ for even *l* and $\eta = -1$ for odd *l*. Then we have

$$v_{0c}(\xi) = (1+(\pi\lambda(\xi)/4\xi)^2)^{1/2} , \tag{26}$$

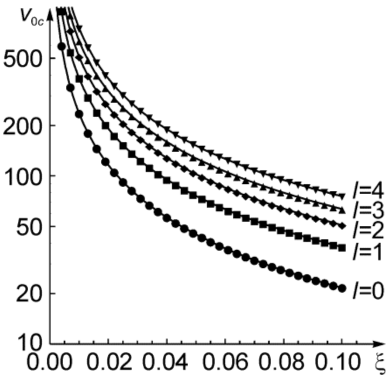

Fig. 2. Dependence of v_{0c} on ξ at ε = 0 and indicated l values for the narrow well.

where $\lambda(\xi)$ is a slowly varying function of variable ξ.

Numerical analysis (see Table 1) for $l = 0, 1, 2, 3, 4, \text{and } 5$ demonstrated that the assumption about slow variations of the function $\lambda(\xi)$ was obeyed with sufficiently good accuracy for any l. In Table 1, v_{0c} was calculated by numerical solution of Eq. (16), and the corresponding λ values were calculated from Eq. (26). As to the assumption about the possibility of application of asymptotic representations for the functions used to solve this problem, its noncontradictory character follows from Table 1, though it is justified with low accuracy.

c) Level ε = –1. In this case, Eq. (7) assumes the form

$$\left(\frac{v_0}{v_0-2}\right)^{1/2}\frac{J_{|l|}(k'\xi)}{J_{|l|+1}(k'\xi)}=0, \tag{27}$$

and for the critical well depth of arbitrary width we obtain for any l

$$v_{0c}=(1+(s_l/\xi)^2)^{1/2}+1, \tag{28}$$

from which it is easy to obtain the corresponding values in the limiting cases of narrow and wide wells.

3. We now analyze how the well depth influences the position of the energy level near the threshold one given by Eq. (28). Let the level $\varepsilon=-1+x$ for $l=0$ appear in the well with depth $v_0=v_{0c}-v$; in this case, $x \ll 1$ and $v \ll v_{0c}$. Substituting these values of energy and well depth into Eq. (7), considering the terms principal with respect to x and v, and designating

$$\alpha=\frac{s_0/\xi}{\left(1+(s_0/\xi)^2\right)^{1/2}}\left(\frac{\left(1+(s_0/\xi)^2\right)^{1/2}-1}{\left(1+(s_0/\xi)^2\right)^{1/2}+1}\right)^{1/2},$$

we obtain

$$x=v+\alpha(x^2)^{1/2}\ln((x/2)^{1/2}\gamma\xi). \tag{29}$$

TABLE 1. Dependence of v_{0c} on ξ at $\varepsilon = 0$ and Indicated l Values for the Wide Well. Values of λ and Arguments of the Bessel Functions Are Also Presented Here

l	ξ	v_{0c}	λ	$k'\cdot\xi$	l	ξ	v_{0c}	λ	$k'\cdot\xi$
0	10	1.02599	2.92171	2.2947	3	10	1.17413	7.83403	6.15283
	100	1.00029	3.04673	2.39289		100	1.00201	8.08382	6.34902
	200	1.00007	3.05429	2.39884		200	1.00051	8.1034	6.3644
	300	1.00003	3.05683	2.40083		300	1.00023	8.11004	6.36961
	400	1.00002	3.0581	2.40183		400	1.00013	8.11338	6.37223
	500	1.00001	3.05886	2.40242		500	1.00008	8.11539	6.37381
1	10	1.06517	4.67115	3.66872	4	10	1.24067	9.34993	7.34342
	100	1.00073	4.8546	3.81279		100	1.00285	9.61488	7.55151
	200	1.00018	4.86656	3.82219		200	1.00072	9.63797	7.56964
	300	1.00008	4.87058	3.82535		300	1.00032	9.64583	7.57581
	400	1.00005	4.8726	3.82693		400	1.00018	9.64978	7.57892
	500	1.00003	4.87381	3.82788		500	1.00011	9.65217	7.5808
2	10	1.11514	6.28325	4.93485	5	10	1.31348	10.843	8.51605
	100	1.0013	6.50678	5.11041		100	1.0038	11.1143	8.72916
	200	1.00033	6.52268	5.1229		200	1.00096	11.1408	8.74992
	300	1.00015	6.52804	5.12711		300	1.00043	11.1498	8.75703
	400	1.00008	6.53074	5.12923		400	1.00024	11.1544	8.76061
	500	1.00005	6.53236	5.1305		500	1.00015	11.1571	8.76277

Solving Eq. (29) by the iteration method and choosing $x_0 = v$ as a zero approximation, in the first approximation we have

$$x_1 = x = \frac{v}{1-\alpha\ln\left(\left(v/2\right)^{1/2}\gamma\xi\right)}\,. \tag{30}$$

Comparing x values calculated from Eq. (30) with numerical solution of exact equation (7), we note that they are in good qualitative agreement for $x << 1$, $v << v_{0c}$, and not too wide well (see Table 2). As expected, the agreement deteriorates with increasing ξ and v.

For $l \neq 0$, considering terms linear in x and v, we have

$$x = \frac{1}{1+\left(\left(v_{0c}-2\right)/v_{0c}\right)^{1/2}\left(s_l/l\right)\xi}\,v\,. \tag{31}$$

The most interesting situation arises when the well depth exceeds the critical one. Here we have the principal difference for states with $l = 0$ and $l \neq 0$. For $l = 0$, a quasi-stationary energy level appears already in the approximation in which only terms linear in x and v are considered; as to the states with $l \neq 0$, the linear approximation appears insufficient for the detection of these levels. These problems will be considered in more details in our future work.

4. We now sum up the results of this work. The critical well depth has been found for the relativistic fermion in the two-dimensional spherically symmetric potential well for energy $E = m$, 0, and $-m$ and zero and nonzero orbital momenta. The approximate analytical expressions derived for this parameter coincided with good accuracy with the results of numerical calculations. For $E = m(1-x)$ $(x << 1)$, the formula for energy of the level was obtained that coincided with the corresponding expression for the nonrelativistic value in the shallow well. The behavior of the level $E = -m(1-x)$ near the threshold was investigated for $l = 0$ and $l \neq 0$. It was demonstrated that it differs insignificantly

TABLE 2. Dependence of the Approximate (x_{ap}) and Exact (x_{ex}) x on v for the Indicated ξ Values

v	ξ = 0.001		ξ = 0.01		ξ = 0.1		ξ = 1	
	x_{ap}	x_{ex}	x_{ap}	x_{ex}	x_{ap}	x_{ex}	x_{ap}	x_{ex}
0.01	0.001002	0.000894	0.001307	0.001146	0.001926	0.001651	0.004392	0.003856
0.02	0.002077	0.001848	0.002738	0.002392	0.004115	0.003510	0.009694	0.008443
0.03	0.003182	0.002828	0.004224	0.003682	0.006429	0.005468	0.015479	0.013386
0.04	0.004309	0.003825	0.005748	0.005002	0.008833	0.007494	0.021628	0.018580
0.05	0.005452	0.004835	0.007301	0.006347	0.011308	0.009576	0.028079	0.023971
0.06	0.006608	0.005857	0.008879	0.007711	0.013844	0.011704	0.034792	0.029522
0.07	0.007775	0.006887	0.010478	0.009091	0.016431	0.013871	0.041740	0.035210
0.08	0.008952	0.007925	0.012096	0.010486	0.019064	0.016073	0.048902	0.041017
0.09	0.010138	0.008971	0.013729	0.011894	0.021740	0.018306	0.056262	0.046929
0.10	0.011331	0.010023	0.015378	0.013313	0.024454	0.020568	0.063808	0.052934

from the spatial case [12]: whereas at $l = 0$ a linear dependence $x \sim v$ was observed in the above-indicated case, a weak logarithmic deviation from this dependence was observed in our case. For $l \neq 0$, no logarithmic term was present in the examined approximation, and the dependence $x(v)$ was linear in character.

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