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STATIC POTENTIAL OF A POINT CHARGE IN REDUCED QED_{3+1}

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The potential of a point source placed on a flat surface is calculated in the context of reduced QED_{3+1}, and the effective charge behavior is investigated with allowance for the polarization of vacuum. Both approximate analytical and numerical methods are used in calculations. It is established that the behavior of the examined potential at short and long distances from the source does not deviate significantly from the Coulomb behavior of vacuum massless and massive fermions. Other deviations of the results obtained from the well-known standard QED_{3+1} and QED_{2+1} data are also discussed.

Keywords: static potential, point charge, surface, polarization of vacuum, vacuum fermion, photon propagator, effective charge.

1. Great interest has been demonstrated recently in quantum field theory on the surface enclosed in space with higher dimensionality [1, 2]. On the other hand, the dynamics of two- and three-dimensional world can be considered as the corresponding dynamics on the surface enclosed in realistic three- and four-dimensional worlds [3, 4]. The term *reduced* here means that while the gauge field that transfers interactions among fermions is propagated in a real volume, the fermions remain localized on the surface.

Enhanced interest in reduced QED_{3+1} has been explained in detail in [3, 4]. In addition, there exists a natural object described by this theoretical model, namely, graphite in which quasi-particles corresponding to elementary fermion excitations – electrons and holes – are localized on two-dimensional surfaces, whereas the electromagnetic field that transfers interactions among them is, generally speaking, a four-dimensional object [5, 6]. Since $v_F << c$ (v_F is the Fermi velocity) in a condensed medium, static (Coulomb) forces here play the leading role in the process of fermion interaction. In this regard, it is expedient to derive an expression for the static potential of point charge field with maximum possible accuracy; in particular, in the present work it is derived with allowance for vacuum loop contribution. This problem has long been solved in [7] for conventional QED_{3+1} and in [8] for QED_{2+1}.

2. As is well known, external current $J_\mu(x)$ creates the field whose potential $A_\mu(x)$ is [7]

$$A_\mu(x) = i\int G^{(\gamma)}_{\mu\nu}(x-x')J_\nu(x')d^4x', \tag{1}$$

where $G^{(\gamma)}_{\mu\nu}(x-x')$ is the total photon propagator. In the examined case, the current assumes the form

$$J_\nu(x') = iQ\delta_{\nu 4}\delta(\boldsymbol{r}'_\perp)\delta(x'_3 - c), \tag{2}$$

where c is a constant. Then for the static potential we obtain

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$$A_0(r_\perp, x_3) = \frac{iQ}{(2\pi)^3}\int \exp(i(\boldsymbol{k}_\perp(\boldsymbol{r}_\perp - \boldsymbol{r}'_\perp) + k_3(x_3 - c)))$$
$$\times G^{(\gamma)}(k_\perp, k_3, k_0 = 0)d^2\boldsymbol{r}'_\perp d^2\boldsymbol{k}_\perp dk_3 \delta(\boldsymbol{r}'_\perp), \tag{3}$$

where Q is the source charge, $G^{(\gamma)}$ is the transverse component of the photon propagator at $k_0 = 0$, and vectors $\boldsymbol{a}_\perp$ have projections $\boldsymbol{a}_\perp(a_1, a_2, 0)$. The reduced photon propagator is determined by the relation

$$G^{(\gamma)(\text{red})}(\boldsymbol{k}_\perp, k_0 = 0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G^{(\gamma)}(\boldsymbol{k}_\perp, k_3, k_0 = 0)dk_3 \tag{4}$$

(below we omit the indication of zero time projections of the Lorentz vectors in arguments of the corresponding functions). Then for the potential A_0 on the plane $x_3 = c$ we obtain

$$A_0(r_\perp, x_3 = c) = \frac{iQ}{(2\pi)^2}\int \exp(i\boldsymbol{k}_\perp \boldsymbol{r}_\perp)G^{(\gamma)(\text{red})}(\boldsymbol{k}_\perp)d^2\boldsymbol{k}_\perp . \tag{5}$$

It can be easily demonstrated that

$$G^{(\gamma)(\text{red})}(\boldsymbol{k}_\perp) = \int G^{(\gamma)}(\boldsymbol{r}_\perp, x_3 = 0)\exp(-i\boldsymbol{k}_\perp \boldsymbol{r}_\perp)d^2\boldsymbol{r}_\perp , \tag{6}$$

that is, $G^{(\gamma)(\text{red})}(\boldsymbol{k}_\perp)$ is the Fourier transform of the photon propagator $G^{(\gamma)}(\boldsymbol{r}_\perp, x_3)$ on the plane $x_3 = 0$. The total reduced propagator is then calculated from the standard formula [3]

$$G^{(\gamma)(\text{red})} = 1/(G_0^{(\gamma)(\text{red})-1} - \Pi), \tag{7}$$

where $G_0^{(\gamma)(\text{red})}$ is the reduced propagator of a free photon and Π is the polarization operator.

We now determine $G_0^{(\gamma)(\text{red})}$ and Π. Substituting expression $G_0^{(\gamma)}(\boldsymbol{k}_\perp, k_3) = \left[i(\boldsymbol{k}_\perp^2 + k_3^2)\right]^{-1}$ into Eq. (4), we obtain

$$G_0^{(\gamma)(\text{red})}(k_\perp, 0) = (2ik_\perp)^{-1} . \tag{8}$$

As to the polarization operator, its standard formula derived in QED_{2+1} [9] can be used if we neglect radiative corrections for the vertex and the fermion propagator (this possibility was indicated in [3]). Then for the reduced photon propagator we obtain

$$G^{(\gamma)(\text{red})}(k_\perp) = \left[2i\left(k_\perp + (e_0^2/8\pi)\left(2m + \frac{k_\perp^2 - 4m^2}{k_\perp}\arctan\frac{k_\perp}{2m}\right)\right)\right]^{-1}, \tag{9}$$

where e_0^2 is the dimensionless coupling constant in reduced QED_{3+1} (in contrast with QED_{2+1}, where it is dimensional) and m is the vacuum fermion mass.

3. We now proceed to calculations of the potential. Integrating Eq. (5) over the angle, we obtain

$$A_0(r_\perp, x_3 = c) = \frac{iQ}{2\pi}\int_0^{\infty} k_\perp J_0(k_\perp r_\perp)G^{(\gamma)(\text{red})}(k_\perp)dk_\perp , \tag{10}$$

where J_0 is the zero-order Bessel function. The indication of fixed plane $x_3 = c$ is omitted below.

We first consider the zero-order approximation of perturbation theory. Then after integration of Eq. (10), we obtain for $G^{(\gamma)(\mathrm{red})}$ given by Eq. (8) [5]

$$A_0(r_\perp) = \frac{Q}{4\pi r_\perp}. \tag{11}$$

The cases of $m = 0$ and $m \neq 0$ are examined separately below (see [8, 10]).

a) The case of $m = 0$. Here $G^{(\gamma)(\mathrm{red})}(k_\perp) = [2ik_\perp(1 + \pi\alpha/2)]^{-1}$, and for the potential we obtain

$$A_0(r_\perp) = \frac{Q}{4\pi(1 + \pi\alpha/2) r_\perp}, \tag{12}$$

where $\alpha = e_0^2/8\pi$. Thus, the fermion vacuum in the examined case behaves as a uniform isotropic dielectric with respect to the point charge field.

b) The case of $m \neq 0$. Substitution of Eq. (9) into Eq. (10) yields an integral which cannot be expressed through the known elementary or special functions. Therefore, the problem is solved by means of approximation of propagator (9) by a rational function that coincides with the propagator at points $k_\perp = 0, k_\perp \to \infty$, and $k_\perp = 2m$. The function

$$G^{(\gamma)(\mathrm{red})}(k_\perp) = \frac{1}{2ik_\perp} \frac{\pi - 2 + k_\perp/m}{\pi - 2 + (1 + \pi\alpha/2)k_\perp/m} \tag{13}$$

meets these conditions.

Functions (9) and (13) are plotted in Fig. 1 from which it can be seen that the employed approximation is in fairly good agreement with exact values of propagator (9) for a wide range of variations of the argument and different coupling constants.

Substitution of Eq. (13) into Eq. (10) and subsequent integration yields

$$A_0(r_\perp) = \frac{Q(r_\perp)}{4\pi r_\perp}. \tag{14}$$

Here we have used the following designations:

$$Q(r_\perp) = \frac{Q}{1 + (\pi\alpha/2)}\left(1 + \left(\pi^2\alpha\beta a/4\right)\left(\boldsymbol{H}_0(\beta a) - Y_0(\beta a)\right)\right), \tag{15}$$

where $Q(r_\perp)$ is the effective charge, $\beta = \dfrac{\pi - 2}{1 + (\pi\alpha/2)}$, $a = mr_\perp$, and $\boldsymbol{H}_0$ and Y_0 are the zero-order Struve and Neumann functions, respectively. Figure 2 shows the dependence of the effective charge on the dimensionless parameter $a = mr_\perp$ calculated from formula (13) and with the use of exact propagator (9). The figure demonstrates that the suggested approximation reproduces sufficiently accurately the behavior of the examined potential.

4. Expanding functions $\boldsymbol{H}_0$ and Y_0 in series for $\beta a \ll 1$ and $r_\perp \ll 1/\beta m$, we can write the effective charge in the form

$$Q(r_\perp) = \frac{Q}{1 + (\pi\alpha/2)}\left(1 - (\pi\alpha\beta a/2)\ln\frac{\beta a}{2}\right). \tag{16}$$

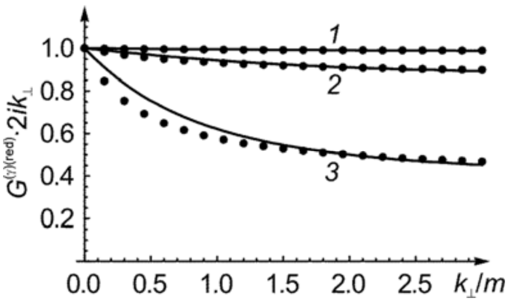

Fig. 1

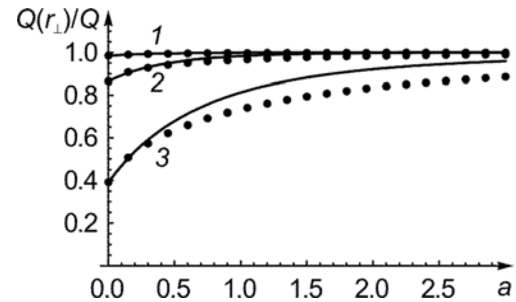

Fig. 2

Fig. 1. Reduced photon propagator calculated from formulas (9) (solid curves) and (13) (closed circles) for $\alpha = 0.01$ (curve *1*), 0.1 (curve *2*), and 1 (curve *3*).

Fig. 2. Dependence of the effective charge on a calculated with exact propagator (9) (solid curves) and from formula (13) (closed circles) for $\alpha = 0.01$ (curve *1*), 0.1 (curve *2*), and 1 (curve *3*).

On the other hand, considering for $\beta a >> 1$ the first two terms of the asymptotic expansion in the difference $\boldsymbol{H}_0 - Y_0$, we have

$$Q(r_\perp) = \frac{Q}{(1+\pi\alpha/2)}\left(1+(\pi\alpha/2)\left(1-\frac{1}{(\beta a)^2}\right)\right). \tag{17}$$

In the limiting cases, we obtain

$$Q(r_\perp) = \frac{Q}{1+(\pi\alpha/2)} \quad (\beta a \to 0)\,, \tag{18}$$

$$Q(r_\perp) = Q \quad (\beta a \to \infty)\,. \tag{19}$$

5. Analyzing Eqs. (12) and (16)–(19) and Fig. 2, we can conclude the following.

1) The effective charge is independent of $r_\perp$ when $m = 0$. This circumstance seems natural, because there are no parameters with dimensionality of length in the examined model, which allows the fermion vacuum to be considered as a uniform dielectric. The vacuum fermion mass here distorts the uniformity of the dielectric. In this regard, this model differs from the QED_3 and QED_4 models. Thus, for the massless QED_3 models, the source charge is somehow screened, and the confinement effect is observed for fermions possessing masses [8, 10, 11] (recall that in this case, the model has the parameter with dimensionality of length – the coupling constant); for QED_4 models with $m = 0$, the source charge can be screened completely. On the other hand, we note that for $m = 0$ and $r_\perp \to \infty$, the limiting value of the effective charge is smaller than for the massive fermions, which is in qualitative agreement with the behavior of the examined quantity in QED_3.

2) For $m = 0$, the effective charge decreases with distance [see Eq. (12)] as the coupling constant α increases. For $m \neq 0$, the effective charge decreases at short distances, whereas its value at long distances tends to Q irrespective of the coupling constant (Fig. 2).

3) For small βa, the effective charge has a logarithmic singularity (it is easily seen directly from Eq. (10) without invoking the approximation procedure). Nevertheless, this singularity does not change the Coulomb character of the potential at short distances $r_\perp$. In this regard, the situation differs radically from standard QED_4 [7].

4) The effective charge at long distances appears greater than at short distances. This differs radically from the well-known cases in QED_3 [8, 9] and QED_4 [7] and somehow reminds the situation in asymptotically free theory [12]. At the same time, the analogy with this theory is incomplete here, since the effective charge in this model is nonzero at $r_\perp = 0$; moreover, the gauge bosons have no colors. On the other hand, this circumstance only vaguely resembles the confinement effect understood as an increase in the potential with distance without single-particle *in-* and *out-states*.

5) The behavior of the correction for the effective charge connected with the polarization of vacuum in the examined case of long distances $(\beta a >> 1)$ is analogous to that in nonlinear QED_4 [7] with a small difference in the distance exponent.

We note that some of the above-listed conclusions can be the artifact of the employed approximation by virtue of consideration of only limited class of diagrams (vacuum loops). Therefore, we believe that the photon propagator for the examined model must be assigned more accurately.

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